# Chaos Driven Differential Evolution In The Task Of Evolutionary Control Of Delayed Logistic Chaotic System

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**Abstract.** In this paper, Differential Evolution (DE) is used for the evolutionary optimization of control of chaotic delayed logistic system. The novality of the approach is that the identical selected discrete dissipative chaotic system is used as the chaotic pseudo random number generator to drive the mutation and crossover process in the DE. The optimization was performed for two types of case studies and developed cost functions.

**Keywords:** Chaos Control, Differential evolution, Optimization, Evolutionary algorithms.
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## INTRODUCTION

These days the methods based on soft computing such as neural networks, evolutionary algorithms, fuzzy logic, and genetic programming are known as powerful tool for almost any difficult and complex optimization problem.

The interest about the interconnection between evolutionary techniques and control of chaotic systems is spread daily. First steps were done in [1], [2], [3] where the control law was based on Pyragas method: Extended delay feedback control – ETDAS [4], [5], [6]. These papers were concerned to tune several parameters inside the control technique for chaotic system. The big advantage of the Pyragas method for evolutionary computation is the amount of accessible control parameters, which can be easily tuned by means of evolutionary algorithms (EA).

This paper is aimed at investigating the chaos driven Differential Evolution (DE). Although a number of DE variants have been recently developed, the focus of this paper is the embedding of chaotic systems in the form of chaos pseudo random number generator (CPRNG) for DE and its application to optimization of chaos control.

Firstly, the problem design is proposed. The next sections are focused on the description of used cost functions, evolutionary algorithm DE and the concept of chaos driven DE. Results and conclusion follow afterwards.

## MOTIVATION

This research is a continuation of the previous successful initial application based experiment with chaos driven DE [7], [8].

This paper extends the research of evolutionary chaos control optimization [3] and initial experiment with chaos driven DE [9].

In this paper the DE/rand/1/bin strategy driven by delayed logistic chaotic map (system) was utilized to solve the issue of evolutionary optimization of chaos control for the very same chaotic system. Thus the idea was to utilize the hidden chaotic dynamics in pseudo random sequences given by chaotic delayed logistic system to help Differential evolution algorithm in searching for the best controller settings for the very same chaotic system.

Recent research in chaos driven heuristics has been fueled with the predisposition that unlike stochastic approaches, a chaotic approach is able to bypass local optima stagnation. This one clause is of deep importance to evolutionary algorithms. A chaotic approach generally uses the chaotic map in the place of a pseudo random number generator [10]. This causes the heuristic to map unique regions, since the chaotic map iterates to new regions. The task is then to select a very good chaotic map as the CPRNG.

Several papers have been recently focused on the connection of DE and chaotic dynamics either in the form of hybridizing of DE with chaotic searching algorithm [11] or in the form of chaotic mutation factor and dynamically changing weighting and crossover factor in self-adaptive chaos differential evolution (SACDE) [12].

The focus of this paper is the embedding of chaotic systems in the form of chaos pseudo random number generator for DE.

The chaotic systems of interest are discrete dissipative chaotic systems. The delayed logistic chaotic system was selected as the CPRNG for DE based on the successful results obtained with DE [13] or PSO algorithm [14].

## PROBLEM DESIGN

The brief description of used chaotic system and original feedback chaos control method ETDAS [4] is given here.

### Selected Chaotic System

The chosen example of discrete dissipative chaotic system used both as a CPRNG and within the evolutionary optimization of chaos control problem was the one-dimensional delayed logistic system.

The delayed logistic is a simple two-dimensional discrete system similar to the one-dimensional Logistic Equation. The map equations are given in (1). The parameter used in this work is $A$ = 2.27 as suggested in [15]. For this value, the system exhibits chaotic behaviour. The example of this behaviour is depicted in numerical simulation of direct system output ($x$ or $y$) in the uncontrolled state (Fig. 1) and $x$-$y$ plot (Fig. 2).

$$\begin{aligned} X_{n+1} &= AX_n(1-Y_n) \\ Y_{n+1} &= X_n \end{aligned} \tag{1}$$

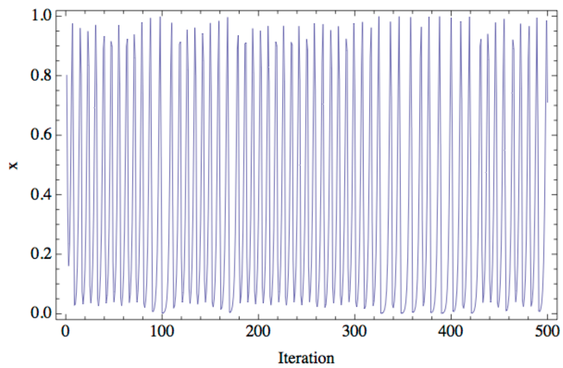


**FIGURE 1.** Direct system output of Delayed logistic system.

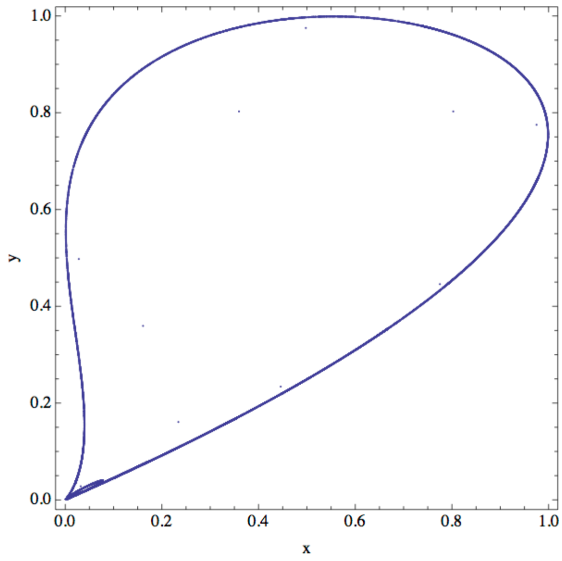


**FIGURE 2.** $x$-$y$ plot of Delayed logistic system.

### ETDAS Control Method

This work is focused on the utilization of the chaos driven DE for tuning of parameters for ETDAS control method to stabilize desired Unstable Periodic Orbits (UPO). In the described research, desired UPO was p-1 (stable state). The original control method – ETDAS in the discrete form suitable for delayed logistic has the form (2).

$$\begin{aligned} x_{n+1} &= Ax_n(1-y_n)+F_n \\ F_n &= K[(1-R)S_{n-m}-x_n] \\ S_n &= x_n+RS_{n-m} \end{aligned} \tag{2}$$

Where: $K$ and $R$ are adjustable constants, which have to be evolutionary tuned. $F$ is the perturbation; $S$ is given by a delay equation utilizing previous states of the system, $m$ is the period of m-periodic orbit to be stabilized. The perturbation $F_n$ in equations (2) may have arbitrarily large value, which can cause diverging of the system outside the output interval of delayed logistic system {0, 1}. Therefore, $F_n$ should have a value between $<-F_{\max}, F_{\max}>$. The suitable $F_{\max}$ value was also obtained from evolutionary optimization process.

## COST FUNCTIONS

This research utilizes and compares two cost function design.

The proposal of the first basic cost function (CF) is in general based on the simplest CF, which could be used problem-free only for the stabilization of p-1 orbit. The idea was to minimize the area created by the difference between the required state and the real system output on the whole simulation interval – $\tau_i$. The simple CF is given in (3).

$$CF_{SIMPLE} = \sum_{t=0}^{\tau_i} \left| TS_t - AS_t \right| \tag{3}$$

Nevertheless this simple approach has one big disadvantage, which is the including of initial chaotic transient behavior of not stabilized system into the cost function value. As a result of this, the very tiny change of control method setting for extremely sensitive chaotic system causing very small change of CF value, can be suppressed by the above-mentioned including of initial chaotic transient behavior.

Another universal cost function had to be used for securing the stabilization of either p-1 orbit (stable state) or higher periodic orbit and having the possibility of adding penalization rules. It was synthesized from the simple CF and other terms were added.

This CF is in general based on searching for desired stabilized periodic orbit and thereafter calculation of the difference between desired and found actual periodic orbit on the short time interval - $\tau_s$ (approx. 20 - 50 iterations) from the point, where the first min. value of difference between desired and actual system output is found (i.e. floating window for minimization). The $CF_{UNI}$ has the form (4).

$$CF_{UNI} = pen_1 + \sum_{t=\tau 1}^{\tau 2} \left| TS_t - AS_t \right| \tag{4}$$

Where:
TS - target state
AS - actual state
$\tau_1$ - the first minimal value of difference between TS and AS
$\tau_2$ – the end of optimization interval ($\tau_1 + \tau_s$)
$pen_1 = 0$ if $\tau_i - \tau_2 \geq \tau_s$;
$pen_1 = 10*(\tau_i - \tau_2)$ if $\tau_i - \tau_2 < \tau_s$ (i.e. late stabilization)

## DIFFERENTIAL EVOLUTION

DE is a population-based optimization method that works on real-number-coded individuals [16]. For each individual $\vec{x}_{i,G}$ in the current generation G, DE generates a new trial individual $\vec{x}'_{i,G}$ by adding the weighted difference between two randomly selected individuals $\vec{x}_{r1,G}$ and $\vec{x}_{r2,G}$ to a randomly selected third individual $\vec{x}_{r3,G}$. The resulting individual $\vec{x}'_{i,G}$ is crossed-over with the original individual $\vec{x}_{i,G}$. The fitness of the resulting individual, referred to as a perturbed vector $\vec{u}_{i,G+1}$, is then compared with the fitness of $\vec{x}_{i,G}$. If the fitness of $\vec{u}_{i,G+1}$ is greater than the fitness of $\vec{x}_{i,G}$, then $\vec{x}_{i,G}$ is replaced with $\vec{u}_{i,G+1}$; otherwise, $\vec{x}_{i,G}$ remains in the population as $\vec{x}_{i,G+1}$. DE is quite robust, fast, and effective, with global optimization ability. It does not require the objective function to be differentiable, and it works well even with noisy and time-dependent objective functions. Description of used DERand1Bin strategy is presented in (5). Please refer to [16], [17], [18] and [19] for the description of all other strategies.

$$u_{i,G+1} = x_{r1,G} + F \cdot \left( x_{r2,G} - x_{r3,G} \right) \tag{5}$$

## CHAOS DRIVEN DE

The main principle of this concept is the embedding of chaotic systems in the form of chaos pseudo random number generator (CPRNG) for DE. In this research, direct output iterations of the chaotic map were used for the generation of real numbers in the process of crossover based on the user defined CR value and for the generation of the integer values used for selection of individuals. The initial concept of embedding chaotic dynamics into the evolutionary algorithms is given in [20].

## EXPERIMENTAL RESULTS

Within the research a total number of 50 simulations with chaos driven DE by means of delayed logistic system were carried out for each CF design. All simulations were successful and have given new optimal settings for ETDAS control method securing the fast stabilization of the chaotic system at required behaviour (p-1 orbit).

Following Tables 2 and 4 contains the simple statistical overview of optimization/simulation results. Tables 3 and 5 contain the best founded individual solutions of parameters set up for ETDAS control method, corresponding final CF value, also the Istab. Value representing the number of iterations required for stabilization on desired UPO and further the average error between desired output value and real system output from the last 20 iterations.

Graphical simulation outputs of the best individual solutions for both case studies are depicted in Fig. 3 and Fig. 5, whereas the Fig. 4 and Fig 6 shows the simulation output of all 50 runs of CHAOS DE, thus confirm the robustness of this approach.

Simulations are depicted only for the variable $x$ of the chaotic delayed logistic system, since the variable $y$ is only time shifted by one iteration (See definition given in chapter 3.1 and (1)).

Settings of EA parameters for both processes were based on performed numerous experiments with chaotic systems (Table 1).

Based on the mathematical analysis, the real p-1 UPO for unperturbed delayed logistic system has following value: $x_S = 0.559471$.

The ranges of all estimated parameters were these:

$-2 \leq K \leq 2$ , $0 \leq F_{max} \leq 0.9$ and $0 \leq R \leq 0.99$

**TABLE 1.** CHAOS DE settings.

| Parameter | Value |
|---|---|
| PopSize | 25 |
| F | 0.8 |
| CR | 0.8 |
| Generations | 500 |
| Max. CF Evaluations (CFE) | 7500 |

## Case study 1 – Simple cost function

From the results presented in the Tables 2 and 3, it follows that the CF-simple is very convenient for evolutionary process, which means that repeated runs of EA are giving identical optimal results (i.e. very close to the possible global extreme). This is graphically confirmed in the Figure 4 when all 50 simulations are basically merged into the one line.

On the other hand the disadvantage of including of initial chaotic transient behavior of not stabilized system into the cost function value and resulting very tiny change of control method setting for extremely sensitive chaotic system is causing suppression of stabilization speed and numerical precision.

**TABLE 2.** CF-simple values statistic

| Statistical data | CF Value |
|---|---|
| Min | 0.199798 |
| Max | 0.199798 |
| Average | 0.199798 |
| Median | 0.199798 |
| Std.Dev. | $5.11 \cdot 10^{-16}$ |
| Avg. Stab. (Iteration) | 10 |

**TABLE 3.** Characteristics of the best solution (CF sim.)

| Parameter | Value |
|---|---|
| $K$ | 1.29837 |
| $F_{max}$ | 0.394579 |
| $R$ | 0.01 |
| CF Value | 0.199798 |
| Istab. Value | 10 |
| Avg. error per iteration | $2.22 \cdot 10^{-17}$ |

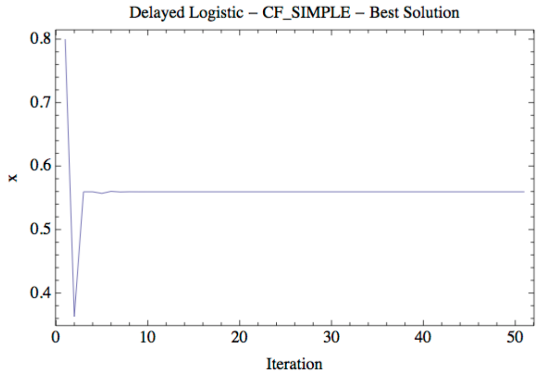


**FIGURE 3.** Simulation of the best individual solution: delayed logistic system - CHAOS DE - CF Simple.

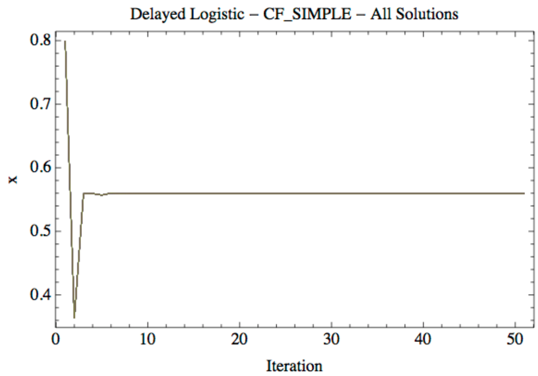


**FIGURE 4.** Simulation of the all 50 solutions: delayed logistic system - CHAOS DE - CF Simple.

## Case study 2 – Universal cost function

Results obtained in this case study lend weight to the argument, that the technique of pure searching for periodic orbits is advantageous for faster and more precise stabilization of chaotic system.

**TABLE 4.** CF-universal simple values statistic

| Statistical data | CF Value |
|---|---|
| Min | $2.3 \cdot 10^{-15}$ |
| Max | $2.7 \cdot 10^{-15}$ |
| Average | $2.44222 \cdot 10^{-15}$ |
| Median | $2.40551 \cdot 10^{-15}$ |
| Std.Dev. | $9.04629 \cdot 10^{-17}$ |
| Avg. Stab. (Iteration) | 7.5 |

**TABLE 5.** Characteristics of the best solution (CF uni)

| Parameter | Value |
|---|---|
| $K$ | 1.31355 |
| $F_{max}$ | 0.336294 |
| $R$ | 0.010219 |
| CF Value | $2.3 \cdot 10^{-15}$ |
| Istab. Value | 8 |
| Avg. error per iteration | 0 |

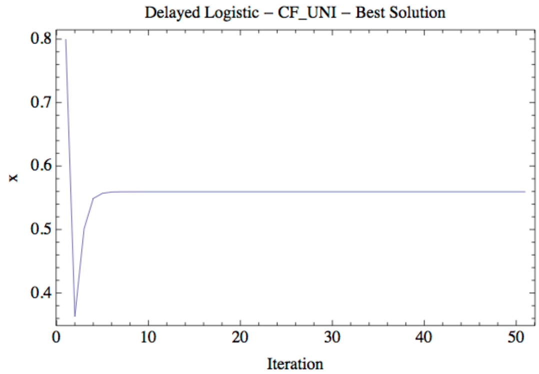


**FIGURE 5.** Simulation of the best individual solution: delayed logistic system - CHAOS DE - CF Universal.

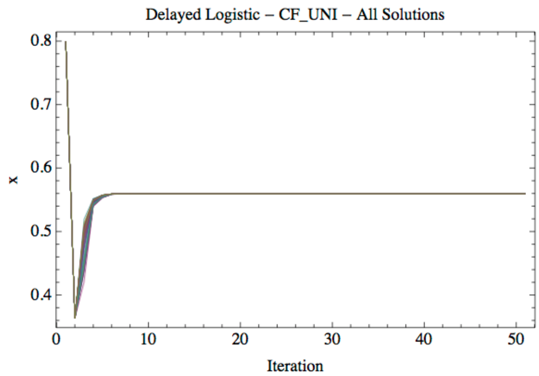


**FIGURE 6.** Simulation of the all 50 solutions: delayed logistic system - CHAOS DE - CF Universal.

## CONCLUSION

Based on obtained results, it may be claimed, that the presented Chaos DE driven by selected discrete dissipative chaotic system has given satisfactory results in the chaos control optimization issue.

The results show that embedding of the chaotic dynamics in the form of chaotic pseudo random number generator into the differential evolution algorithm may help to improve the performance and robustness of the DE. Thus to obtain optimal solutions securing the very fast and precise stabilization for both convenient CF surface in case of the CF-simple and very chaotic and nonlinear CF surface in case of the CF-universal.

When comparing the both CF designs, the CF-simple is very convenient for evolutionary process (i.e. repeated runs are giving identical optimal results), but it has many limitations.

The second universal CF design brings the possibility of using it problem free for any desired behavior of arbitrary chaotic systems, but at the cost of the highly chaotic CF surface. Nevertheless the embedding of the chaotic dynamics into the evolutionary algorithms helped to deal with such an issue.

The primary aim of this work was not to develop any new pseudo random number generator, which should normally pass many statistical tests, but to show that through embedding the hidden chaotic dynamics into the evolutionary process in the form of chaotic pseudo random number generators may help to obtain better results and avoid problems connected with evolutionary computation such as premature convergence and stagnation in local extremes.

Future plans include testing of different chaotic systems, either manually or evolutionary tuning of chaotic maps parameters, comparisons with different heuristics and obtaining a large number of results to perform statistical tests.

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## REFERENCES

1. I. Zelinka, R. Senkerik and E. Navratil, *Chaos, Solitons & Fractals* **40** (1), 111-129 (2009).
2. R. Senkerik, I. Zelinka, D. Davendra and Z. Oplatkova, “Evolutionary Design of Chaos Control in 1D”, in *Evolutionary Algorithms and Chaotic Systems*, edited by I. Zelinka, S. Celikovsky, H. Richter and G. Chen (Springer Berlin Heidelberg, 2010), Vol. 267, pp. 165-190.
3. R. Senkerik, I. Zelinka, D. Davendra and Z. Oplatkova, *Computers & Mathematics with Applications* **60** (4), 1026-1037 (2010).
4. K. Pyragas, *Physics Letters A* **206** (5–6), 323-330 (1995).
5. W. Just, “Principles of Time Delayed Feedback Control”, in *Handbook of Chaos Control*, edited by H. G. Schuster (Wiley-VCH Verlag GmbH & Co. KGaA, 2006), pp. 21-41.
6. K. Pyragas, *Physics Letters A* **170** (6), 421-428 (1992).
7. D. Davendra, I. Zelinka and R. Senkerik, *Computers & Mathematics with Applications* **60** (4), 1088-1104 (2010).

8. R. Senkerik, D. Davendra, I. Zelinka, Z. Oplatkova and M. Pluhacek, "Optimization of the Batch Reactor by Means of Chaos Driven Differential Evolution", in *Soft Computing Models in Industrial and Environmental Applications*, edited by V. Snášel, A. Abraham and E. S. Corchado (Springer Berlin Heidelberg, 2013), Vol. 188, pp. 93-102.
9. R. Senkerik, D. Davendra, I. Zelinka and Z. Oplatkova, "Chaos Driven Differential Evolution in the Task of Chaos Control Optimization", in *2010 IEEE Congress on Evolutionary Computation (CEC)* (2010), pp. 1408-1415.
10. I. Aydin, M. Karakose and E. Akin, *Expert Systems with Applications* **37** (7), 5285-5294 (2010).
11. W. Liang, L. Zhang and M. Wang, *Journal of Software* **6** (7), 1297- 1304 (2011).
12. G. Zhenyu, C. Bo, Y. Min and C. Binggang, „Self-Adaptive Chaos Differential Evolution", in *Advances in Natural Computation*, edited by L. Jiao, L. Wang, X.-b. Gao, J. Liu and F. Wu (Springer Berlin Heidelberg, 2006), Vol. 4221, pp. 972-975.
13. R. Senkerik, M. Pluhacek, Z. K. Oplatkova, D. Davendra and I. Zelinka, "Investigation on the Differential Evolution Driven by Selected Six Chaotic Systems in the Task of Reactor Geometry Optimization", in *2013 IEEE Congress on Evolutionary Computation (CEC)* (2013), pp. 3087-3094.
14. M. Pluhacek, R. Senkerik, D. Davendra, Z. Kominkova Oplatkova and I. Zelinka, *Computers & Mathematics with Applications* **66** (2), 122-134 (2013).
15. J. C. Sprott, *Chaos and Time-Series Analysis*. (Oxford University Press, 2003).
16. R. Storn and K. Price, *Journal of Global Optimization* **11** (4), 341-359 (1997).
17. K. V. Price, "An Introduction to Differential Evolution", in *New Ideas in Optimization*, edited by D. Corne, M. Dorigo and F. Glover (McGraw-Hill Ltd., 1999).
18. K. Price and R. Storn, "Differential evolution homepage" (2001) [Online]. Available: http://www.icsi.berkeley.edu/~storn/code.html
19. K. V. Price, R. M. Storn and J. A. Lampinen, *Differential Evolution - A Practical Approach to Global Optimization*. (Springer Berlin Heidelberg, 2005).
20. R. Caponetto, L. Fortuna, S. Fazzino and M. G. Xibilia, *IEEE Transactions on Evolutionary Computation* **7** (3), 289-304 (2003).