# MATHEMATICAL MODELLING OF THE THERMAL PROCESS IN THE AQUATIC ENVIRONMENT

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This paper presents the mathematical model of the thermal influence to the aquatic environment of thermal power plant, which is solved by the Navier-Stokes and temperature equations for an incompressible fluid in a stratified medium. Numerical algorithm based on the projection method which solved with fractional step method. Three dimensional Poisson equation solved with Fourier method with combination of tridiagonal matrix method (Thomas' algorithm).

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**The "lead paragraph" is encapsulated with the LaTeX `quotation` environment and is formatted as a single paragraph before the first section heading. (The `quotation` environment reverts to its usual meaning after the first sectioning command.) Note that numbered references are allowed in the lead paragraph. The lead paragraph will only be found in an article being prepared for the journal *Chaos*.**

## I. INTRODUCTION

Environment - the basis of human life, as mineral resources and energy are produced from them. Moreover they are the basis of modern civilization. However, the current generation of energy cause appreciable harm to the environment, worsening living conditions. The basis of the same energy - are the various types of power plants. But power generation in thermal power plants (TPP), hydro power plant (HPP) and nuclear power plants (NPP) is associated with adverse effects on the environment. The problem of the interaction of energy and the environment has taken on new features, extending the influence of the vast territory, most of the rivers and lakes, the huge volumes of the atmosphere and hydrosphere. Previously, the impact on the environment TPP was not in first priority, as before to get electricity and heat had a higher priority. Technology of production of electrical energy from power plant is connected with a lot of waste heat released into the environment. Today the problem of influence of the nature by power is particularly acute because the pollution of the atmosphere and hydrosphere increases each year. Another problem, of TPP is thermal pollution of reservoirs or lakes. Dropping hot water - is a push chain reaction that begins reservoir overgrown with algae, it violates the oxygen balance, which in turn is a threat to the life of all its inhabitants. Thermal power plants with cooling water shed 4 - 7 kJ of heat for 1 kW / h electricity generation. Meanwhile, the Health Standards discharges of warm water with TPP should not raise the temperature higher than in the summer and in winter of the reservoir initial temperature. Spread of harmful emissions from TPP depends on several factors: the terrain, environmental temperature, wind speed, cloud cover, precipitation intensity. Speed deployment and increases the thermal pollution area - are meteorology conditions. As seen in figure 1, large proportion of electricity (81.3 %) in the world is produced by thermal power plants. Therefore, emissions of this type of power plants to the atmosphere and hydrosphere, provide the greatest amount of anthropogenic contaminants in it.

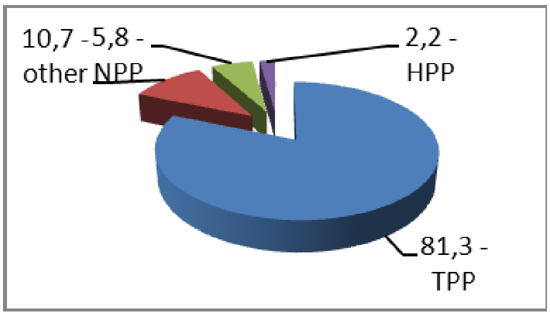


FIG. 1. Electricity production in the world by type of power plant (2010), %

Ekibastuz SDPP -1 is taken as an example of such effects of the TPP to the aquatic environment, located in Pavlodar region in 17 km. To the North-East of the city Ekibastuz, Kazakhstan.

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## II. MATHEMATICAL MODEL

Actually in the pond-coolers spatial temperature is quite low. So stratified flow in the pond-cooler can be described by approaching to the Boussinesq equations. Therefore systems like equations of motion, continuity and temperature are used for mathematical modelling. Moreover well-developed spatial turbulent is considered for stratified pond[1–3]. Tree dimensionally model is used for distribution of temperature modelling in a reservoir[4,5]:

$$\frac{\partial \overline{u_i}}{\partial t}+\frac{\partial \overline{u_j u_i}}{\partial x_j}=-\frac{\partial \overline{p}}{\partial x_i}+\nu\frac{\partial}{\partial x_j}\left(\frac{\partial \overline{u_i}}{\partial x_j}\right)+\beta g_i(T-T_0)-\frac{\partial \tau_{ij}}{\partial x_j} \quad (1)$$

$$\frac{\partial \overline{u_j}}{\partial x_j}=0 \quad (i=1,\,2,\,3). \quad (2)$$

$$\frac{\partial T}{\partial t}+\frac{\partial u_j T}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\chi\frac{\partial T}{\partial x_j}\right) \quad (3)$$

where

$$\tau_{ij}=\overline{u_i u_j}-\overline{u_i u_j} \quad (4)$$

$g_i-$ the gravity acceleration, $\beta-$the coefficient of volume expansion, $u_i$ - velocity components, $\chi-$ thermal diffusivity coefficient , $T_0-$the equilibrium temperature, $T-$deviation of temperature from the balance.

We start with regular LES corresponding to a "bar-filter" of $\Delta x$ width, an operator associating an function $\overline{f}(\overline{x},t)$. We then define a second "test filter" tilde of large width $2\Delta x$ associating $\widetilde{f}(\bar{x},t)$. Let us first apply this filter product to the Navier-Stokes equation. The sub grid-scale tensor of the field $\widetilde{\overline{u_i}}$ is obtained from equation (4) with the replacement of the filter bar by the double filter and tilde filter:

$$\tau_{ij}=\overline{\widetilde{u_i}\widetilde{u_j}}-\overline{\widetilde{u_i u_j}} \quad (5)$$

$$l_{ij}=\overline{\widetilde{u_i}\widetilde{u_j}}-\overline{\widetilde{u_i u_j}} \quad (6)$$

We now apply the tilde filter to equation (4), which leads to

$$\widetilde{\tau_{ij}}=\overline{\widetilde{u_i u_j}}-\overline{\widetilde{u_i u_j}} \quad (7)$$

Adding equations (6) and (7) and using equation (5), we obtain

$$l_{ij}=\tau_{ij}-\widetilde{\tau_{ij}}$$

Now we have to determine , the stress resulting from the filter product. This is again obtained using the Smagorinsky model, which yields to

$$\widetilde{\tau_{ij}}-\frac{1}{3}\delta_{ij}\widetilde{\tau_{kk}}=-2C\widetilde{A_{ij}} where A_{ij}=(\Delta x)^2\left|\overline{S}\right|\overline{S_{ij}} \quad (8)$$

We now have to determine$\tau_{ij}$, the stress resulting from the filter product. This is again obtained using the Smagorinsky model, which yields

$$\tau_{ij}-\frac{1}{3}\delta_{ij}\tau_{kk}=-2CB_{ij} where B_{ij}=(2\Delta x)^2\left|\widetilde{\overline{S}}\right|\widetilde{\overline{S_{ij}}} \quad (9)$$

Subtracting (8) from (9) with the aid of Germano's identity yields to

$$l_{ij}-\frac{1}{3}\delta_{ij}l_{kk}=2CB_{ij}-2C\widetilde{A_{ij}}$$

$$l_{ij}-\frac{1}{3}\delta_{ij}l_{kk}=2CM_{ij}$$

where

$$M_{ij}=B_{ij}-\widetilde{A_{ij}} \quad (10)$$

All the terms of equation (10) may now be determined with the aid of $\overline{u}$. Unfortunately, there are five independent equations for only one variable C, and thus the problem is over determined. A first solution proposed by Germano is to multiply (10) tensor ally by $\overline{S_{ij}}$ to get

$$C=\frac{1}{2}\frac{l_{ij}\overline{S_{ij}}}{M_{ij}\overline{S_{ij}}}$$

This provides finally dynamical evaluation of C, which can be used in the LES of the bar field $\overline{u}$[6].

Initial and boundary conditions are defined for the non-stationary 3D equations of motion, continuity and temperature, satisfying the equations.

## III. NUMERICAL ALGORITHM

Numerical solution of (1) - (3) is carried out on the posted grid using the scheme against a stream of the second type and compact approximation for convective terms[7–10]. Scheme of splitting on physical parameters is used to solve the problem in view of the above with the proposed model of turbulence. It is anticipated that at the first stage the transfer of momentum occurs only through convection and diffusion. Intermediate field of speed is handled by using method of fractional steps through the tridiagonal method (Thomas algorithm)[11].

In the second phase is for pressure which is found by the help of intermediate field of speed. Poisson equation for pressure is solved by Fourier method in combination with the tridiagonal method (Thomas algorithm) that is applied to determine the Fourier coefficients[1,3]. At the third stage, it is supposed that the transfer is carried out only by the pressure gradient. The algorithm was parallelized on the high-performance system[2,3].

$$I)\frac{\vec{u}^{*}-\vec{u}^{n}}{\tau}=-\left(\nabla\vec{u}^{n}\,\vec{u}^{*}-\nu\Delta\vec{u}^{*}\right)$$

$$II)\Delta p=\frac{\nabla\vec{u}^{*}}{\tau}$$

$$III)\frac{\vec{u}^{n+1}-\vec{u}^{*}}{\tau}=-\nabla p.$$

## IV. RESULTS OF COMPUTATIONAL MODELING

Initial and boundary conditions were posed to meet the challenges. In the calculation we used the mesh of 100x100x100 size. Figure 2 shows Ekibastuz SDPP-1 relief area and the estimated spatial path and isolines of the temperature distribution at different points in time after the launch of SDPP-1, on the surface of the water, the side view. Figure 3 shows Ekibastuz SDPP-1 location and the path and isolines of the temperature distribution at different points in time after the launch of SDPP-1, on the surface of the water, the another side view. Figure 4 shows Ekibastuz SDPP-1 location and the path and isolines of the temperature distribution at different points in time after the launch of SDPP-1, on the surface of the water, top view.

## V. CONCLUSION

Thermal distribution with the disposal of runoff is insulated on both diagrams. The results show that the thermal distribution is over a large area. Therefore, well-developed model of three-dimensional stratified turbulent flow makes it possible to identify qualitatively and approximately quantitatively the basic patterns of hydrothermal processes occurring in waters.

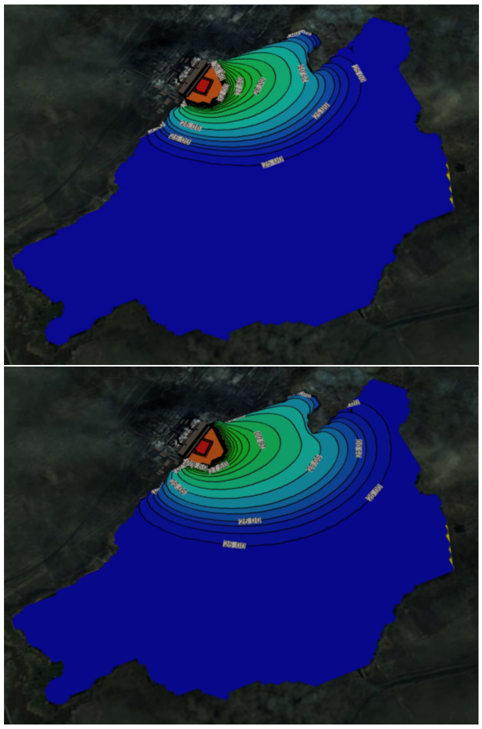

FIG. 2. Outline and isolines of the temperature distribution after 15 and 20 hours after the launch of SDPP-1, on the surface of the water, the side view.

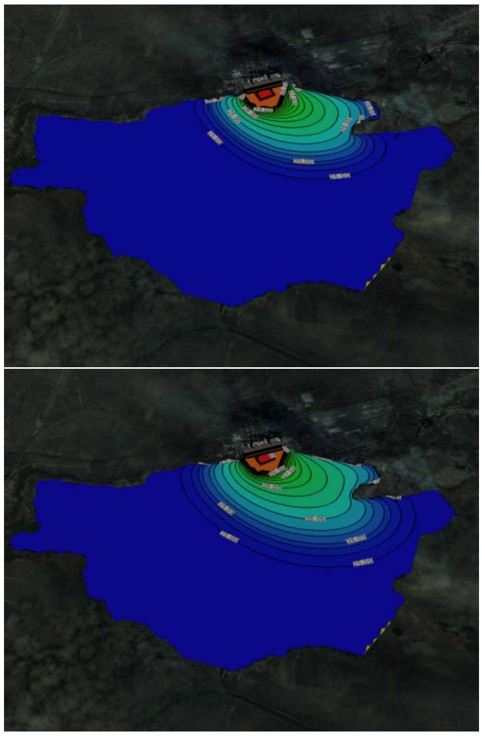

FIG. 3. Outline and isolines of the temperature distribution through 15 and 20 hours after the launch of SDPP-1, on the surface of the water, the side view.

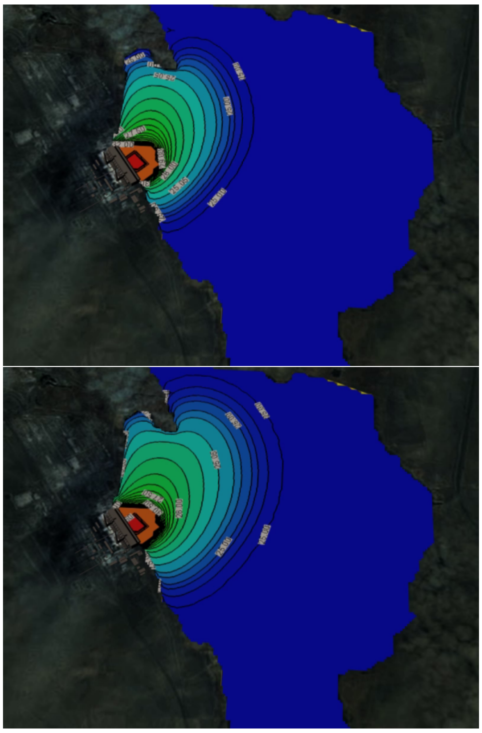

FIG. 4. Outline and isolines of the temperature distribution through 15 and 20 hours after the launch of SDPP-1, on the surface of the water, top view.


[1] A. Issakhov, "Large eddy simulation of turbulent mixing by using 3d decomposition method," J. Phys.: Conf. Ser. 318 **Issue 4**, 042051 (2011).

[2] B. Zhumagulov and A. Issakhov, "Parallel implementation of numerical methods for solving turbulent flows," Vestnik NIA **1(43)**, 12–24 (2012).

[3] A. Issakhov, "Development of parallel algorithm for numerical solution of three-dimensional poisson equation," Journal of Communication and Computer **Volume 9, Number 9**, 977–980 (2012).

[4] A. Issakhov, "Mathematical modelling of the influence of thermal power plant to the aquatic environment by using parallel technologies," AIP Conf. Proc. **Volume 1499**, 15–18 (2012).

[5] A. Issakhov, "Mathematical modelling of the influence of thermal power plant on the aquatic environment with different meteorological condition by using parallel technologies," Power, Control and Optimization. Lecture Notes in Electrical Engineering **Volume 239**, 165–179 (2013).

[6] M. Lesieur, O. Metais, and P. Comte, *Large eddy simulation of turbulence* (New York, Cambridge University Press, 2005).

[7] C. A. Fletcher, *Computational Techniques for Fluid Dynamics. Vol 2: Special Techniques for Differential Flow Categories* (Berlin: Springer-Verlag, 1988).

[8] P. J. Roache, *Computational Fluid Dynamics* (Albuquerque, NM: Hermosa Publications, 1972).

[9] R. Peyret and D. T. Taylor, *Computational Methods for Fluid Flow* (New York: Berlin: Springer-Verlag, 1983).

[10] J. C. Tannehill, D. A. Anderson, and R. H. Pletcher, *Computational Fluid Mechanics and Heat Transfer. 2nd ed.* (New York: McGraw-Hill, 1997).

[11] N. N. Yanenko, *The Method of Fractional Steps* (New York: Springer-Verlag. In J.B.Bunch and D.J. Rose (eds.), Space Matrix Computations, New York: Academics Press, 1979).