

Two Easy To Apply Solution Procedures To The Determination Of Production-shipment Policy

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Abstract. This paper deals with the determination of production-shipment policy for an integrated vendor-buyer system. The inventory model contains two decision variables: the replenishment lot size and the number of deliveries. This paper solves the inventory model considering two cases: 1) the replenishment lot size as continuous variable and the number of shipments as discrete variable, and 2) the replenishment lot size and the number of shipments as discrete variables. Two easy to apply solution procedures are proposed.

Keywords: manufacturing system replenishment lot size, delivery, random defective rate.

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INTRODUCTION

It is well known that the first inventory models EOQ and EPQ were proposed by Harris [1] and Taft [2] respectively. Since then many extensions to the EOQ/EPQ inventory models have been appearing in several journals. See for example the research works of [3-9].

Recently, Chiu et al. [6] present a two-phase algebraic approach to determine the replenishment and the number of shipments for an integrated vendor-buyer system. They basically derive the inventory model using the method of completing perfect squares [7-10]. We read Chiu et al. [6]'s paper with a substantial interest and we found that they considered both (replenishment lot size and number of shipments)

variables as continuous. However, the number of shipments must be considered as discrete variable. We think that the readers interested in this type of inventory problem may be interested in knowing a solution procedure to solve the optimizing problem considering the variables according their nature. In this direction, this paper considers two cases: Case 1) the replenishment lot size (Q) continuous and the number of shipments (n) discrete, and Case 2) the replenishment lot size (Q) discrete and the number of shipments (n) discrete.

MATHEMATICAL MODELLING

We refer to the reader to see in Chiu et al. [6] the assumptions and detailed derivation of the total cost of the inventory system. Chiu et al. [6] derive the following total cost, here rewritten as follows:

$$E[TCU(Q, n)] = \alpha_1 + \left(\alpha_2 + \frac{\alpha_5}{n} \right) Q + \frac{\alpha_3 + \alpha_4 n}{Q} \quad (1)$$

where;

$$\alpha_1 = \frac{C\lambda}{(1-\phi E[x])} + C_r\lambda + \frac{C_R E[x](1-\theta)\lambda}{(1-\phi E[x])} + \frac{C_s E[x]\phi\lambda}{(1-\phi E[x])} > 0 \quad (2)$$

$$\alpha_2 = \frac{\lambda}{(1-\phi E[x])} \left[\frac{h_1(E[x])^2(1-\theta)^2}{2P_1} + \frac{h}{2P} + \frac{h}{2P_1} [2E[x] - (E[x])^2 - \phi(E[x])^2(1-\theta)] \right] \quad (3)$$

$$\left[\frac{h(1-\phi E[x])}{2} - (h-h_2) \left[\frac{\lambda}{2P} + \frac{E[x](1-\theta)\lambda}{2P_1} \right] \right] > 0$$

$$\alpha_3 = \frac{K\lambda}{(1-\phi E[x])} > 0 \quad (4)$$

$$\alpha_4 = \frac{K_1\lambda}{(1-\phi E[x])} > 0 \quad (5)$$

$$\alpha_5 = (h-h_2) \left[-\frac{(1-\phi E[x])}{2} + \frac{\lambda}{2P} + \frac{E[x](1-\theta)\lambda}{2P_1} \right] \quad (6)$$

$$\phi = \theta + (1-\theta)\theta_1 > 0 \quad (7)$$

Where the notation is:

Parameters:

- C : Production cost (\$/unit)
- C_R : Rework cost (\$/unit)
- C_S : Disposal cost per scrap product (\$/unit)
- C_T : Delivery cost per product shipped to customers (\$/unit)
- K : Setup cost per cycle (\$/cycle; \$/lot;\$/run)
- K_1 : Delivery cost per shipment (\$/shipment)
- h : Holding cost (\$/unit/time unit)
- h_1 : Holding cost for each reworked product (\$/unit/time unit)
- h_2 : Holding cost for each item kept by customer (\$/unit/time unit)

- λ : Demand rate (units/time unit)
- P : Production rate (units/time unit)
- P_1 : Reworking rate (units/time unit)
- x : A proportion of defectives; it is assumed to be a random variable with a known probability density.
- $E(x)$: Expected value of x
- θ : A proportion of the imperfect quality products is scrap; it is assumed to be known and constant.
- θ_1 : A proportion of reworked products fails and becomes scrap; it is assumed to be known and constant.
- ϕ : Overall scrap rate per cycle;
 $\phi = \theta + (1-\theta)\theta_1$.
- $U(a, b)$: Indicates uniform distribution with range (a, b)

Variables:

- Q : The replenishment lot size (units)
- n : The number of shipments

Considering Q as continuous variable, using the algebraic method of completing perfect squares [7-10] or differential calculus, it is easy to show that Equation (1) is minimized when

$$Q = \sqrt{\frac{\alpha_3 + \alpha_4 n}{\left(\alpha_2 + \frac{\alpha_5}{n} \right)}} \quad (8)$$

Substituting Equation (8) into Equation (1) then the minimum total cost is;

$$E[TCU(n)] = \alpha_1 + 2\sqrt{f_n + \alpha_2\alpha_3 + \alpha_4\alpha_5} \quad (9)$$

where;

$$f_n = \alpha_2\alpha_4 n + \frac{\alpha_3\alpha_5}{n} \quad (10)$$

According to García-Laguna et al. [11] the function f_n is minimized when;

$$n = \left\lceil -0.5 + \sqrt{0.25 + \frac{\alpha_3\alpha_5}{\alpha_2\alpha_4}} \right\rceil \quad (11)$$

or

$$n = \left\lfloor 0.5 + \sqrt{0.25 + \frac{\alpha_3\alpha_5}{\alpha_2\alpha_4}} \right\rfloor \quad (12)$$

It is important to remember that $\lceil \beta \rceil$ and $\lfloor \beta \rfloor$ are the smallest integer greater than or equal to β , and the

largest integer less than or equal to β , respectively. Undoubtedly, it is easy to see that $\lceil \beta \rceil = \lfloor \beta + 1 \rfloor$ if and only if β is not a discrete value. In this condition the minimization problem has only one solution for n which is $n^* = n$ (given by either of the two expressions in Equations (11) and (12). Otherwise, the minimization problem has two solutions for n that are $n^* = n$ and $n^* = n + 1$. This result was also used in Cárdenas-Barrón [12] for deriving of the close form for calculating the number of shipments for the inventory models of Chang [13] and Lin [14].

It is easy to see that $\alpha_2\alpha_4 > 0$. However, $\alpha_3\alpha_5$ can be positive, zero or negative because the following term α_5 can be positive, zero or negative. When α_5 takes positive values then the optimal solution for n is given by Equation (11) or (12). On the other hand, for zero and negative values of α_5 , it is obvious that f_n attains its global minimum value at $n=1$. Now, if Q is restricted to be a discrete variable then Equation (1) attains its minimum when;

$$Q = \left\lceil -0.5 + \sqrt{0.25 + \frac{\alpha_3 + \alpha_4 n}{\left(\alpha_2 + \frac{\alpha_5}{n}\right)}} \right\rceil \quad (13)$$

or

$$Q = \left\lfloor 0.5 + \sqrt{0.25 + \frac{\alpha_3 + \alpha_4 n}{\left(\alpha_2 + \frac{\alpha_5}{n}\right)}} \right\rfloor \quad (14)$$

A lower bound for the total cost can be derived easily just considering both variables as continuous and it is given by;

$$LB = \alpha_1 + 2\left(\sqrt{\alpha_2\alpha_3} + \sqrt{\alpha_4\alpha_5}\right) \quad (15)$$

It is important to remark that the lower bound for the total cost for Case 2 is the total cost of Case 1.

Taking account the above results Equations (1), (8), (11), (12), (13) and (14) we propose two solution procedures to solving two cases previously mentioned.

Case 1) The Replenishment Lot Size (Q) Continuous And The Number of Shipments (n) Discrete

In this case the appropriated solution procedure is shown in Figure 1.

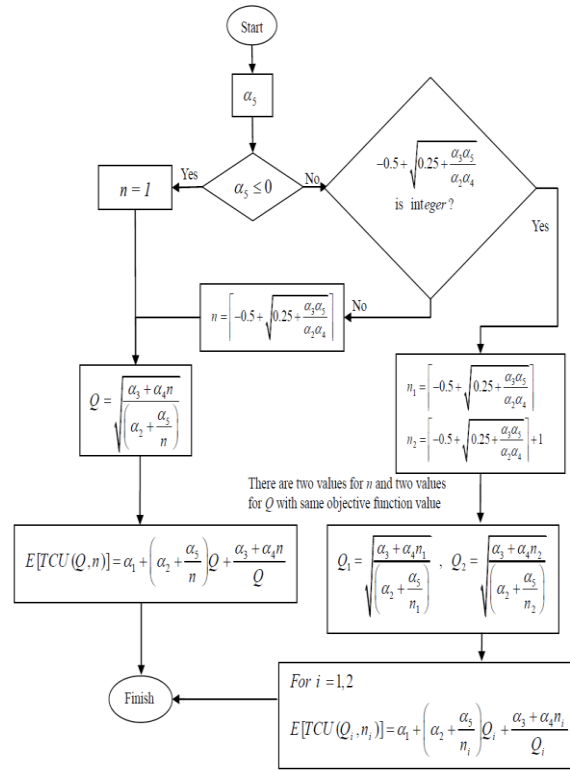


FIGURE 1. The solution procedure for the Case 1.

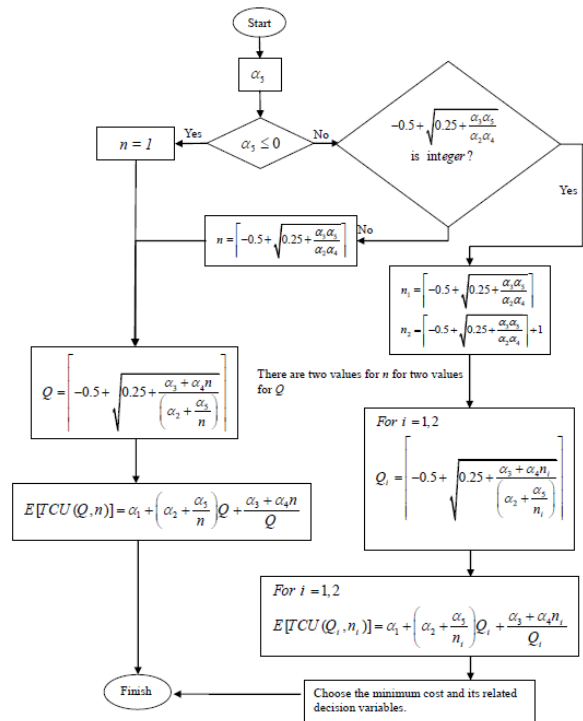


FIGURE 2. The solution procedure for the Case 2.

DISCUSSION

The numerical example given in [6] is solved using the two proposed solutions procedures. The data for the example is shown in Table 1 and the solutions are given in Table 2 and Table 3. Applying the solution procedure for Case 1 we obtain the same solution as Chiu et al. [6] but in a straightforward way (see Table 2). On the other hand, the solution procedure of Chiu et al. [6] requires to evaluate the total cost twice (one for each n) in order to determine the number of shipments (n).

It is important to remark that Chiu et al. [6] do not consider the situation when $\alpha_5 \leq 0$. Also, they do not solve the inventory problem when both variables are discrete. These are important aspects that our paper treated. Our paper simplifies, improves and complements Chiu et al. [6] research work. Finally, the readers interested in this paper may be also interested in the research works of Cárdenas-Barrón et al. [15-17].

TABLE 1. Data for example

Parameter	From Chiu et al. [6]
C	100
C_R	60
C_S	20
C_T	0.1
K	20000
K_1	2000
h	20
h_1	40
h_2	80
λ	3400
P	60000
P_1	2100
x	Uniform(0,0.3)
$E(x)$	0.15*
θ	0.1
θ_1	0.1
$\varphi = \theta + (1 - \theta)\theta_1$	0.19

* Notice that for a uniform distribution with range (a,b) the expected value is defined as $E(x) = \frac{a+b}{2}$

TABLE 2. Results for the example for Case 1

Instance	Q continuous and n discrete Solution procedure 1
Example from Chiu et al. [6]	Q=1735.128997 n=3 E[TCU]=485540.6602929 LB=485540.6485389

TABLE 3. Results for the example for Case 2

Instance	Q and n discrete Solution procedure 2
Example from Chiu et al. [6]	Q=1735 n=3 E[TCU]=485540.6605828 LB=485540.6602929

CONCLUSION

This paper presents two easy to apply solutions procedures to determine jointly both the optimal replenishment lot size and the optimal number of shipments for an integrated vendor-buyer system. The proposed solution procedures are easy and require no tedious computational effort. Furthermore, the proposed solution procedures discriminate between the situation in which there is a single solution and when there are two solutions for each discrete variable.

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