Preventive Maintenance Optimization for Multi States Series-Parallel System

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Abstract. The paper develops a model to evaluate the availability and the cost optimization for a multi-states series-parallel system, whose components are subject to periodic preventive maintenance. The objective is to optimize for each system component the maintenance policy minimizing a cost function of the system, under the constraint of required availability and for a specified period. This policy identifies the good times of components to perform preventive maintenance, as well as the dates of the first inspections.
We consider that system cost is a cost of preventive maintenance. The universal generating function model is used to assess the performance distribution of the entire system and the system availability. The optimization is done for different values of required availability. The effect of required availability on the preventive maintenance policy and system cost is studied. The optimization technique is based on the genetic algorithm. An illustrative example is presented.

Key Words: Genetic Algorithm, Multi-States System, Optimization, Preventive Maintenance, Universal Generating Function.

INTRODUCTION

Many researchers have studied systems maintenance optimization during their lifetime. These studies aim to enhance the system functionality by increasing its expected performance.

The main interest in the industrial world is to find a balance between the system safety and availability, and the system cost. The globalization of the markets and the importance given to the quality management and failure risk, generate an important competition between industries. This leads, on the one hand, to minimize the cost of system function, and on the other hand, to enhance system performance in order to answer the industrial needs. Consequently, the research on the domain of maintenance optimization is an important topic. The reliability allocation involves the design of reliable systems with minimal costs by adopting an applicable and effective methodology. In this family of allocation models, it can be distinguished the approaches based on the reliability maximization under cost constraints, the cost minimization under the constraint of reliability, or the bi-objective ones. Given that modern systems are very dynamic and accessible for interventions, extensive research focus on the topic of optimal maintenance and surveillance policy. Reference [1] presents a study considering different maintenance strategies, reparation and economic constraints.

The evolution of system reliability depends on its structure as well as on the evolution of the reliability of its elements. The latter is a function of the element age on a system's operating life. Element ageing is strongly affected by maintenance activities performed on the system [2].

Preventive maintenance (PM) consists of actions that improve the condition of system elements before they fail. PM actions either return the element to its initial condition and the element becomes 'as good as new', or reduce the age of the element. In some cases, the PM activity does not affect the state of the element but ensures that the element is in operating condition. In this case the element remains 'as bad as old.'

Optimizing the policy of preliminary planned PM actions is the subject of much research activities. In the past, the maintenance problem was often treated in the context of repairing the system failure. Less attention has been given to economic systems where failures are dormant and are only detected by periodic tests or inspections.

This paper develops availability and cost models for systems with periodically inspected and maintained components subjected to periodic maintenance strategy. The aim of our research is to optimize, for each component of a system, the maintenance policy minimizing the cost function, with respect to the availability constraint A_0, and for a given mission time T_M. The first time inspection is determined based on Birnbaum important factor. Component becomes "as good a new" after each inspection. A genetic algorithm (GA) is used as an optimization technique. The best maintenance policy can be found by using the universal generating function to assess the availability of the studied system. The solution comprises both the availability and the cost evaluation.

The universal generating technique was well used to study the reliability of multi-states system [3][4].

A similar optimization problem applied on series–parallel multi-state system was studied in reference [5] taking into account imperfect component PM actions. This model does not consider the first time of inspection, and the age of component is reduced by a factor after each imperfect maintenance action.

The authors in reference [2] have developed a method to determine an optimal periodic maintenance policy in a series-parallel system but not multi-states system. The Monte Carlo simulation was used to assess the system availability.

System description

We adopt a series-parallel multi-states system with non-identical binary independent components. The system maintenance cost is function of the inspection cost of each component. In our study, the cost of inspection for each component is the same for all the mission period. All components are immediately and perfectly repairable after failure. The failure times of each component for a fixed load occur according to an exponential distribution (constant rate).

Once the maintenance period of system component is fixed, one can evaluate the component performance distribution and its maintenance cost. Hence, the system availability, the system performance, and its total maintenance cost are evaluated. The objective function (the system cost) is known. The optimization procedure seeks for the components maintenance periods that maximize the objective function under system availability constraint. Numerical example is considered and the process is applied for different required availability values.

Section 2 describes the PM model for general series–parallel systems. Section 3 formulates the optimization problem. Section 4 presents the universal generating function approach and explains how to use it to evaluate the performance distribution (i.e. the possible system states with their probabilities and their corresponding system performances) in a complex series-parallel system. Section 5 presents the technique used to found the optimal first inspection vector. Section 6 assesses the cost optimization technique. Section 7 illustrates the optimization approach by a numerical application. Section 8 concludes and shows possible extensions.

PREVENTIVE MAINTENANCE MODEL FOR GENERAL SERIES-PARALLEL SYSTEM

Maintenance model for basic components

We assume that the PM actions improve the reliability of basic component to as good as new, thus the component's age is restored to zero. The problem to find the optimal vector T_p is closely connected with another problem, i.e. to find the optimal first inspection time vector T_0: because, it makes no sense to carry out inspections in the beginning of the life of a system, when both the system and its basic components are very reliable [2]. Thus the optimal vector T_0 must be found for each of basic components. The optimal vector T_0 must be constructed so that it takes into account both cost and reliability view.

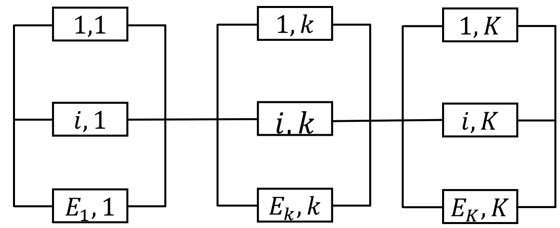

FIGURE 1. General series-parallel structure

General series–parallel structure

In this paper, we adopt a series-parallel system structure that is shown in Fig. 1.

K is the number of series sub-systems, and E_K is the number of parallel components in the k^{th} series sub-system.

Cost model

The cost of the presented preventive maintenance policy of a series-parallel system can be calculated as a function of the inspections costs of the system components. One can write:

$$C_{PM} = \sum_{k=1}^{K}\sum_{i=1}^{E_k}\sum_{l=1}^{\eta_{e(i,k)}} c_l(e(i,k)) \quad (1)$$

$\eta_{e(i,k)}$ is the number of total inspections of the i^{th} component in the k^{th} series sub-system (parallel bloc) during the mission time. $c_l\big(e(i,k)\big)$ is the cost of the l^{th} inspection of the i^{th} component in the k^{th} series sub-system. K is the number of series sub-systems, and E_K is the number of parallel components in the k^{th} series sub-system. $N = \sum_{k=1}^{K}\sum_{i=1}^{E_k} e(i,k)$ is the number of system components. We assume that the inspection cost of the component is constant during the mission time. Hence one can write:

$$C_{PM}\big(e(i,k)\big) = \sum_{l=1}^{\eta_{e(i,k)}} c_l\big(e(i,k)\big) = \eta_{e(i,k)} * c\big(e(i,k)\big) \quad (2)$$

$c\big(e(i,k)\big)$ is the cost inspection of the i^{th} component in the k^{th} series sub-system, $\eta_{e(i,k)} = 1 + \left\lfloor \frac{T_M(e(i,k)) - T_0(e(i,k))}{T_p(e(i,k))} \right\rfloor$

T_M, T_0, and T_p are respectively the mission time, the time of the first inspection and the maintenance period of the component.

PROBLEM FORMULATION

The system is composed of many subsystems connected in series. Each subsystem contains different components connected in parallel. Each component *j*, is characterized by its failure rate $\lambda_j(t)$, and PM cost of one inspection: $c\big(e(i,k)\big)$ (i is the number of the j^{th} component in the series subsystem *k*).

Maintenance actions or inspections are carried out periodically for j^{th} basic component with the period of $T_p(j)$. Inspections are perfect, which means that the component is renewed-model as good as new is assumed. The inspection of the j^{th} component begins at the time $T_0(j)$.

The time in which a component is not available due to PM activity is negligible, if compared to the time elapsed between consecutive activities. Components are supposed to be binary but the entire system is a multi-states system.

The objective of this study is to optimize for each system component, the maintenance policy minimizing a cost function C_{PM} and respecting the availability constraint ($A(t) \geq A_0,\ \forall t, 0 < t \leq T_M$). Thus we have to find optimal cost minimizing vectors $T_p = [T_p(1),\ T_p(2), \dots, T_p(N)]$ and $T_0 = [T_0(1),\ T_0(2), \dots, T_0(N)]$ under given availability constraint A_0. The problem can be formulated with Equation (3).

$$\begin{cases} C_{PM} \rightarrow Min \\ A(t) \geq A_0 \\ t \leq T_M \end{cases} \tag{3}$$

The determination of the T_0 vector is based on the Birnbaum importance factor of system components, evaluated in the context of multi-states. The component maintenance cost is function of its inspection cost, the maintenance period and the date of the first inspection.

Having the above assumptions, and for each maintenance policy, the performance distribution of each component can be deduced and thus the probability mass function of the entire MSS is obtained. Then the system availability, and the corresponding maintenance cost can be obtained.

AVAILABILITY CALCULATION BASED ON UNIVERSAL GENERATING FUNCTION

In a Multi-State System (MSS), each state is characterized by a performance level. In this paper, the distribution that defines the possible system states and their corresponding performance levels is called performance distribution.

Many methods have been used to evaluate the reliability and performance distribution in a MSS such as the Monte-Carlo simulation, the extension of the Boolean model to multi-valued case, the Markov process etc..., [1][3][6][7] but all of them are overworked and time consuming when each system component might have different states. On the contrary, the Universal Generating Function (UGF) technique is fast enough to treat this type of systems [8][9]. It allows finding the entire MSS performance distribution based on the performance distribution of its components by using a fast algebraic procedure. According to the generic MSS model, any system component j can have n_{gj} different states corresponding to the performance levels, represented by the set $G_j = \{G_{j1},\dots,G_{jn_{gj}}\}$ [4]. The UGF consists on attributing for each system component j having the productivity distribution vector G_j the following function:

$$U_j(z) = \sum_{l=1}^{n_{gj}} P_{jl}\, z^{G_{jl}}$$

n_{gj} is the states number for the component *j*, P_{jl} is the probability that the component *j* is at the state *l*, and G_{jl} is the corresponding productivity. In order to obtain the UGF of a subset composed by a certain number of components, the composition operators are used. These operators determine the resulting function *U*(*z*) of a group of series-parallel system based on simple algebraic operations. Equation (4) and Equation (5) give the resultant UGF of the combination of two components 1 and 2:

$$U(z) = \Omega(U_1(z), U_2(z)) = \Omega\Big(\sum_{l=1}^{n_{g1}} \alpha_l z^{a_l}, \sum_{k=1}^{n_{g2}} \beta_k z^{b_k}\Big) \tag{4}$$

$$U(z) = \sum_{l=1}^{n_{g1}} \sum_{k=1}^{n_{g2}} \alpha_l \beta_k z^{w(a_l, b_k)} \tag{5}$$

$w(a_l, b_k)$ is the equivalent productivity of the two components; for a series components $w(a_l, b_k) = \min(a_l, b_k)$, and for a parallel components $w(a_l, b_k) = a_l + b_k$ [8][9].

Thus, to evaluate the performance distribution of the entire system, each connected pair (when two system components have a functional relation between their performance level due to their parallel, series or other connections, they are said to be connected pair), is replaced with an equivalent component, with the *U*-function obtained, and the process is repeated if the rest of system contains more than one component.

Our multi-states system is composed by binary components. The performance of any failed component is equal to zero, and the performance of component in working state is G_j.The UGF representing the probability mass function of the component performance can be written as: $u_j(z) = (1 - A_j)z^0 + A_j z^{G_j}$

Once the performance distribution of each component is obtained, compositions operators can be applied to obtain the probability mass function of the entire system.

We suppose that the failure distribution of the component j follows the exponential law, hence its availability can be written as:

$$A_j(t) = exp\left(-\frac{t}{\lambda(j)}\right) = exp\left(-\frac{t}{MTTF(j)}\right) \quad (6)$$

Each component j is subject to ideal preventive maintenance. The maintenance period is $T_p(j)$. Hence the asymptotic component availability is:

$$A_j = exp\left(-\frac{T_p(j)}{MTTF(j)}\right)$$

Equation (7) gives the resultant UGF of the combination of two components 1 and 2:

$$\mathrm{U(z)} = \Omega\big(\mathrm{U}_1(z), \mathrm{U}_2(z)\big) = \Omega\left(\sum_{i=1}^{2} \alpha_i \mathrm{Z}^{a_i}, \sum_{j=1}^{2} \beta_j \mathrm{Z}^{b_j}\right) = \sum_{i=1}^{1}\sum_{j=1}^{2} \alpha_i \beta_j \mathrm{Z}^{w(a_i, b_j)} \quad (7)$$

$\alpha_1 = 1 - A_1, a_1 = 0, \alpha_2 = A_1, a_2 = G_1,\ \beta_1 = 1 - A_2, a_1 = 0, \alpha_2 = A_2, a_2 = G_2$.

OPTIMAL FIRST INSPECTION VECTOR T_0

Naturally, the problem of finding the optimal vector T_p is closely connected with another problem, namely the finding of vector T_0 which represents the beginning of inspections of each basic component, i.e. the vector of first inspection times [2]. Inspections in the beginning of the life of a component, when the component is very reliable are not efficient. Consequently, we must find the optimal vector T_0. The first intervention into the system must be effective from both reliability and cost point of view.

The inspection is more efficient when its cost is low and when the system increase in availability due to this inspection is more important. In our study, we use the time dependent ratio-criterion of efficiency that is defined as:

$$R_j(t) = \frac{c(j)}{IFB_j(t)}, j = 1,2, \ldots, N$$

N is the number of system components. $IFB_j(t)$ is the Birnbaum's measure of importance of j^{th} component at time t (e.g. definition in reference [10]).

The objective is to minimize this criterion for each system component. Because the inspection cost is constant for each component, minimizing $R_j(t)$ is achieved by maximizing $IFB_j(t)$. Hence the objective is to find optimal $T_0(j)$ that maximize the Birnbaum's measure of importance for each component j, $\mathrm{j} = 1,2, \ldots, \mathrm{N}$.

Consider a vector of M maintenance times $t_m = [t_m(1), t_m(2), \ldots, t_m(M)]$. We evaluate for each maintenance time $t_m(i), i = 1, \ldots, M$, the corresponding importance factor $IFB_j\big(t_m(i)\big)$ for each component j based on Equation (8).

$$IFB_j\big(t_m(i)\big) = \frac{\Delta E}{\Delta P_j} = \frac{E_{2j} - E_{1j}}{P_{2j} - P_{1j}} \quad (8)$$

$E_{2j} - E_{1j}$ is the system availability variation due to the transition of component j from the state of availability P_{1j} to that of availability P_{2j}. We can, for example, consider the transition from the failure state to the normal performance state. E_{2j}, and E_{1j} are evaluated. Thus ΔE and $IFB_j\big(t_m(i)\big)$ can be deduced. The following procedure determines the optimal vector $T_0 = [T_0(1),\ T_0(2), \ldots, T_0(N)]$.

For each inspection time $\mathrm{t_m(i)}, i = 1, \ldots M$, the importance factor is evaluated for all system components.

Step 1

For each component $j, j = 1,2, \ldots, N$, evaluate its availability $A_j\big(t_m(i)\big) = exp\left(-\frac{t_m(i)}{MTTF(j)}\right)$.

Step2

For $j=1$, calculate the system availability when the component j is down $A_j\big(t_m(i)\big) = 0$, by using the UGF approach.

Step 3

Recalculate the system availability when the component j is functioning: $A_j\big(t_m(i)\big) = 1$.

Step 4

Deduce the Birnbaum importance factor $IFB_j\big(t_m(i)\big)$ of the component j at the time $t_m(i)$.

Step 5

$j = j + 1, j \leq N$, return to the step 2.

Step 6

$i = i + 1, i \leq M$, return to the step 1.

We obtain for each component j, a vector of importance factor $IFB_j = [IFB_j(t_m(1)), \ldots IFB_j(t_m(M))]$. We choose the date of the first inspection of the component j, $T_0(j)$ corresponding to $IFB_j(t_m(i))$ maximal. Then the optimal vector $T_0 = [T_0(1), \ldots, T_0(N)]$ can be deduced.

COST OPTIMIZATION TECHNIQUE

Several universal optimization techniques have been designed through the years. References [11][12][13] present a study about optimization techniques applied in the field of system reliability. The authors have distinguished between two classes of optimization: direct methods or exact resolutions; they guarantee the optimality in a finite time, as Dynamic programming, Implicit enumeration, Separation and evaluation (branch and bound) , Lexicographic research, etc., and indirect methods; they are used when there is no sufficient information, the global optimum is not always guaranteed, for example, Heuristics, Meta-heuristics, Genetic algorithms, Tabu search, Simulated annealing, Ant colony, etc.

Our problem is characterized by a large space of solutions. As the objective function (the quality of the solution) is the only available information, the resolution should be done through meta-heuristic methods. The genetic algorithm is one of the most powerful meta-heuristic methods. It has been adopted to solve many reliability optimization problems. It is inspired from the genetic biology. It is based on the principle of evolutionary search.

The solutions are represented by chromosomes in the form of strings. Any maintenance policy can be represented as an *N*-length integer string x in which any element $x_j (0 \le x_j \le T_M)$ determines the maintenance period for system component j.

The adopted genetic algorithm can be described by the following steps:

1. Built an initial population of N_s solutions generated randomly.
2. Evaluate each chromosome in the population.
3. Obtain new solutions by using crossover, and mutations with probability p_m .The crossover facilitates the inheritance of some basic proprieties from the parents by the offspring, and mutation maintains a diversity of solutions. This procedure avoids convergence to a local optimum, and facilitates jumps in the solution space [14].
4. Decode the string and evaluate the solution corresponding to the obtained maintenance policy (determine the system availability and the preventive maintenance cost).
5. The objective value is used to compare different solutions. The solutions are ordered from the best to the worst. The best solutions join the population, and the other ones are discarded.
6. Repeat N_r times steps 2 to 5.

NUMERICAL APPLICATION

Our optimization problem is applied on the system adopted by [15]. It is a power plant multi-stage coal feeding system with nine conveyors. Each conveyor is characterized by a constant failure rate λ_0 and by a fixed cost of preventive maintenance action $c(e(i,k))$, i is the number of the component in the series subsystem *K*. The mean time of failure of the components follows the exponential law. The mission time is $T_M = 50\ years$. The structure of the system is presented in Fig 2.

The parameters λ_0 and $c(e(i,k)$ for each component are given in Table 1.

The first inspection should not be too late, because it will be followed by a periodic preventive maintenance.

We decide to choose this time between 1 and 20 years.

7.

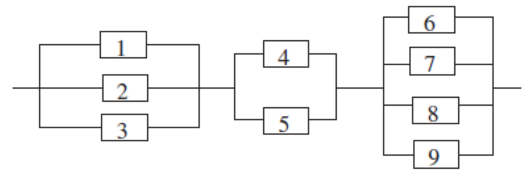

FIGURE 2. Block diagram of the coal feeding system

TABLE 1. Parameters of the Conveyors in the Coal Feeding System

Number of Element	$\lambda_0(y^{-1})$	$c(e(i,k)$
1	0,0692	6,92
2	0,1005	8,04
3	0,1229	9,83
4	0,0383	7,66
5	0,0383	7,66
6	0,1203	9,63
7	0,1203	9,63
8	0,0929	11,15
9	0,0929	11,15

We discretized the time and we generate twenty times of maintenance $t_m(i), i = 1, \ldots, 20$. Then the Birnbaum importance factor $IFB_j(t_m(i))$ for each component j and for each maintenance time is evaluated. The maximal value of $IFB_j(t_m(i))$ for each component is saved. Then we get the optimal first inspection vector $T_0 = [12\ \ 13\ \ 13\ \ 9\ \ 9\ \ 15\ \ 15\ \ 14\ \ 14]$.

After getting the optimal vector T_0, the optimal vector T_p is determined. The corresponding preventive maintenance cost and system availability are deduced.

The optimization problem is treated under the availability constraint $A_0 = 0.9$ in the first time. Many tests have been realized with different values of parameters ($N_S, N_r, et\ p_m$: population size, generation number, and probability of mutation respectively). The best solution were obtained for $N_S = 100$, $N_r = 2000$, and $p_m = 0,05$. The result is presented in the Table 2.

The quasi optimal solution is obtained for $p_m = 0.05$. The optimal vector of maintenance period is:

$$T_P = [3.9503\quad 4.8774\quad 4.7992\quad 4.2542\quad 5.1262\quad 5.1912\quad 5.9838\quad 7.9376\quad 3.5357].$$

Many run of GAs with different required system availability have been done: $A_0 = 0.9$, $A_0 = 0.85$, $A_0 = 0.8$, $A_0 = 0.75$, $A_0 = 0.7$, and $A_0 = 0.6$. The effect of this factor on the optimal preventive maintenance cost is studied. Results are summarized in the Table 3.

<u>$\boldsymbol{A_0 = 0.9}$</u>

T_P
$= [3.9503 \quad 4.8774 \quad 4.7992 \quad 4.2542 \quad 5.1262 \quad 5.1912$
$5.9838 \quad 7.9376 \quad 3.5357]$
$C_{PM} = 654$
$\underline{\boldsymbol{A_0 = 0.85}}$
$T_P = [3.3478 \quad 3.8303 \quad 38.7443 \quad 5.3897 \quad 5.5135$
$11.59 \quad 4.7023 \quad 7.2354 \quad 4.5155]$
$C_{PM} = 556$
$\underline{\boldsymbol{A_0 = 0.8}}$
T_P
$= [3.828 \quad 4.4121 \quad 21.1873 \quad 4.6277 \quad 8.6873 \quad 20.4044$
$4.7849 \quad 8.0540 \quad 6.0395]$

TABLE 2. Preventive maintenance cost for different values of probability of mutation $\mathbf{p_m}$

$\mathbf{p_m}$	.01	.02	.03	.04	.05	.06	.07	.08	.09
$\mathbf{C_{PM}}$	740	692	744	706	654	730	670	738	726

.1	.2	.3	.4	.5	.6	.7	.8	.9	1
709	746	695	730	698	731	673	680	687	690

TABLE3. Preventive maintenance cost for different values of $\mathbf{A_0}$

$\mathbf{A_0}$	.9	.85	.8	.75	.7	.6
$\mathbf{C_{PM}}$	654	556	487	444	410	341

$C_{PM} = 487$
$\underline{\boldsymbol{A_0 = 0.75}}$
T_P
$= [4.0512 \quad 8.4623 \quad 10.3868 \quad 6.1312 \quad 8.3814 \quad 5.1852$
$6.1601 \quad 11.3836 \quad 12.0384]$
$C_{PM} = 444$
$\underline{\boldsymbol{A_0 = 0.7}}$
T_P
$= [3.5028 \quad 15.8973 \quad 13.6606 \quad 10.2995 \quad 6.9699 \quad 7.4332$
$4.5909 \quad 6.054 \quad 41.6296]$
$C_{PM} = 410$
$\underline{\boldsymbol{A_0 = 0.6}}$
T_P
$= [7.7378 \quad 9.3785 \quad 6.397 \quad 11.1022 \quad 11.7438 \quad 7.3279$
$9.9306 \quad 20.9482 \quad 9.0426]$
$C_{PM} = 341$

It can be seen that, the cost of the optimal maintenance policy decreases quickly when the required availability decreases. This is due to the fact that the number of preventive maintenance actions of components required to answer the availability constraints decreases (components can be less available). Hence, the cost of preventive maintenance of system components, and the entire system decrease. This fact can be verified by observing the maintenance periods of system components. These periods increase (number of maintenance decreases) when the required availability decreases.

It can be seen also that as well as the system availability increases, the cost of the optimal maintenance policy becomes more sensitive to the required availability (the variation of this cost increases with the required availability). Increasing the system availability when working with high required availability need high maintenance cost.

CONCLUSIONS AND PERSPECTIVES

This paper shows the efficiency of an optimization method to minimize the PM cost of series–parallel systems based on the time dependent Birnbaum importance factor and using universal generating function and GAs. The effect of the required availability on the preventive maintenance cost is also studied. Varying required performance can allow directing the optimization more or less in the way of component loading.

The presented method can be extended to more complex systems, viz. no exponential failure rates, complex structures different than series–parallel ones, dependent, etc.

Introduction of Markov process in the described approach is possible and it allows the study of optimization problem for a finite horizon time (at a given point of time), or even to integrate more type of dependencies between system elements. The process can increase significantly the needed computing time for solving the problem, but hybridization of the genetic algorithm with other optimization methods like local optimization can reduce this time.

Also, more investigation to study the mathematical properties of the objective function and a complexity analysis should help to improve the solving procedure and compare the performance of the different optimization methods.

ACKNOWLEDGEMENTS

The authors are grateful to the Lebanese CNRS, and to the cedar project for their financial support.

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