

# Energy Function Analysis Based on Variable Gradient Method for a Three-Machine Infinite-Bus Power System

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**Abstract.** The construction of Lyapunov functions for electric power systems is a challenging task. There is no general method for obtaining these functions. Lately, several methods which arise from Lyapunov's second method have been proposed and used to construct a Lyapunov function for a given system. In this study, we focus on providing a suitable candidate Lyapunov (energy) function by using the variable gradient method. The used method is based upon Lyapunov's second method. A three-machine infinite-bus power system with the governors and automatic voltage regulators removed is used to validate the used method performance. Simulations show that the variable gradient method provides high-performance in studied power system.

**Keywords:** Electric power system, Lyapunov (energy) function, Lyapunov's second method, Variable gradient method

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## INTRODUCTION

Recently, a great deal of attention has been received and a fairly large body of literature has been focused on the stability of electric power systems. Power system stability is one of the most important problems in large and heavily loaded interconnected systems with a wide range of devices. A power system at a given initial operating state and subjected to a given disturbance is stable if the system state stays within determined bounds and the system comes into a new stable state of equilibrium within a determined period of time [1]. The stability of electric power systems is categorized into transient, steady-state and dynamic stability. The problem of transient stability has been explained by examining solutions of systems of differential equations based on the swing equation or driven pendulum [2].

Lyapunov defined his direct method (also called Lyapunov's second method), which is a very useful one based on the energy notion [3]. If energy of a system is continuously decreasing until an equilibrium state is reached, the system is stable. If the total energy of a system is continuously decreased, the time derivative of Lyapunov (also known as energy) function is negative definite [4]. The energy function is a vital tool for the stability analysis of power systems.

The most important handicap is that there are no general methods for constructing a Lyapunov function for a given system. Some approaches to constructing Lyapunov functions are suitable. One of them is the variable gradient method, which provides a formal

Lyapunov function design. The present paper deals with the variable gradient method.

A new way of constructing Lyapunov functions has been outlined, called the variable gradient method by Schultz and Gibson in [5]. See also [6], where authors used same method for the construction of a Lyapunov function. Both papers indicate that the variable gradient method is a logical and systematic technique for constructing Lyapunov functions by Lyapunov's second method. DeMarco [7] described a new method of constructing Lyapunov functions for "quasi-gradient" systems. Recently, the variable gradient method has successfully been applied to electric power systems [8-10].

In [11], Kopell and Washburn used Melnikov's method for detecting chaotic motions in the two-degree-of-freedom swing equations. Salam and et al. [12] presented a simulation of a two machine power system, each nonzero damping. Ueda et al. [2] studied coupled swing equations modeling the dynamics of two electric generators connected to an infinite bus by a simple transmission network. They concluded that the attractor-basin phase portrait provides the most direct information about system stability. The concepts of transiently chaotic swings and windowed Lyapunov exponents to power system dynamics has been introduced by Liu et al. in [13], the proposed method being applied to a three machine system with lossless transmission lines therein. The two degrees of freedom swing equation system has been investigated as the model of electric power system to analyze its transient stability problem, and chaotic behavior and global basin structure have also been studied in [14, 15]. Okuno et al. [16] examined OGY (Ott-Grebogi-Yorke)

method for controlling chaos of three machines operating onto an infinite bus system. See also [17], where authors investigated same power system for controlling by using a delayed feedback method.

The aim of the paper is to provide a suitable candidate Lyapunov (energy) function by using the variable gradient method based on Lyapunov's direct method. The proposed method is applied to three-machine infinite-bus power system. The simulation results for the studied model are obtained using MAPLE [18] software.

The organization of this paper is as follows: A three-machine infinite-bus power system model is given in section 2. In section 3, outlines the main idea of the variable gradient method for constructing a suitable Lyapunov function. In section 4, gives a candidate Lyapunov (energy) function of the studied model. Numerical simulation is given to verify the effectiveness of the energy function using MAPLE software in section 5. Finally, conclusions are stated in section 6.

## POWER SYSTEM MODEL

Consider the power system shown in Fig. 1 that three-synchronous-machine is connected to an infinite bus through a transmission line with the governors (GOV) and automatic voltage regulators (AVR) removed [17, 19].

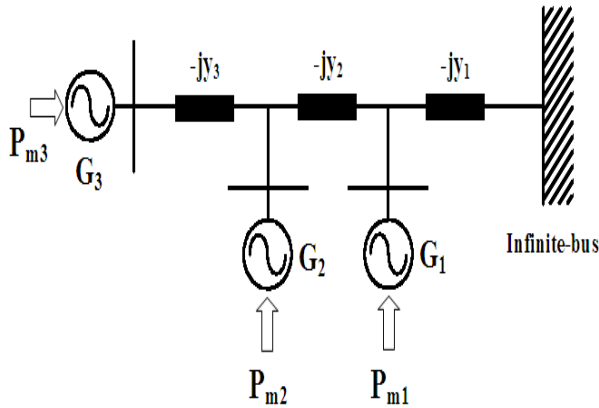


FIGURE 1. Power system model [19]

The three-machine infinite-bus power system can be mathematically described by the following pair of differential equations:

$$\left. \begin{aligned} \dot{\delta}_1 &= \omega_1 \\ \dot{\omega}_1 &= \frac{1}{M_1} \{-D_1\omega_1 + P_1 - K_1V_1 \sin \delta_1 - K_2 \sin(\delta_1 - \delta_2)\} \\ \dot{\delta}_2 &= \omega_2 \\ \dot{\omega}_2 &= \frac{1}{M_2} \{-D_2\omega_2 + P_2 - K_2 \sin(\delta_2 - \delta_1) - V_2 \sin(\delta_2 - \delta_3)\} \\ \dot{\delta}_3 &= \omega_3 \\ \dot{\omega}_3 &= \frac{1}{M_3} \{-D_3\omega_3 + P_3 - V_2 \sin(\delta_3 - \delta_2)\} \end{aligned} \right\} (1)$$

where  $\delta_i$  is the rotor angle of the  $i$ th machine with respect to a synchronously rotating reference frame,  $\omega_i$  is the angular speed of the  $i$ th machine,  $P_{mi}$  is the mechanical input power,  $-jy_i$  is admittance of transmission line negligible resistance,  $M_i$  is inertia constant of the  $i$ th machine and  $D_i$  is damping coefficient of the  $i$ th machine. For the detailed system equations of this model, see reference [17].

## THE VARIABLE GRADIENT METHOD

This section starts by reviewing the usual variable gradient method [8-10]. Consider the nonlinear system

$$\dot{x} = f(x, t) \quad (2)$$

Denote a test Lyapunov function by using  $V$  and its time derivative  $\dot{V}$ . In (2),  $V$  is  $x$ 's open function but not  $t$ 's.

$$\dot{V} = \frac{\partial V}{\partial x_1} \dot{x}_1 + \frac{\partial V}{\partial x_2} \dot{x}_2 + \dots + \frac{\partial V}{\partial x_n} \dot{x}_n \quad (3)$$

Denoted from the point of the gradient, equation (3) can be written as:

$$\dot{V} = (\nabla V)^* \dot{x} \quad (4)$$

where  $(\nabla V)^*$  is the transpose of  $\nabla V$ .  $V$  is obtained as a line integral of  $\nabla V$  [20], as:

$$V = \int_0^x (\nabla V)^* dx \quad (5)$$

The integral's upper limit here does not point that  $V$  is a vector quantity, but integral is preferred to line integral of a random point  $(x_1, x_2, \dots, x_n)$  at the phase-space. This integral can be made independent of integration method.

A gradient system of the form [7]

$$\dot{x} = -A\nabla V(x, x_0) \quad (6)$$

$V: \mathfrak{R}^n \times \mathfrak{R}^n \rightarrow \mathfrak{R}$  is a continuously differentiable function.  $A \in \mathfrak{R}^{n \times n}$  is defined as  $\det(A) \neq 0$  and  $V(x, x_0) = 0$  at  $x = x_0$ .

If the Hessian of  $V(x, x_0)$  is completely positive definite at  $x_0$ , the equilibrium point is asymptotically stable at  $x_0$ .

Lyapunov function is given as:

$$V(x) = \int_{x_0}^x [f(\xi)] d\xi \quad (7)$$

The equation (7) will be used in order to construct a suitable candidate Lyapunov function of the three-machine infinite-bus power system.

### ENERGY FUNCTION FOR THE THREE-MACHINE INFINITE BUS POWER SYSTEM

The three-machine infinite-bus power system model differential equations above can be written again under the condition that the generator mechanical power is equivalent to the active load requirement ( $P_m = P_l$ ).

$$\dot{\delta}_1 = \frac{1}{M_1} M_1 \omega_1 \quad (8)$$

$$\dot{\omega}_1 = -\frac{D_1}{M_1^2} M_1 \omega_1 - \frac{1}{M_1} f(\delta_1, \delta_2, \delta_3) \quad (9)$$

$$\dot{\delta}_2 = \frac{1}{M_2} M_2 \omega_2 \quad (10)$$

$$\dot{\omega}_2 = -\frac{D_2}{M_2^2} M_2 \omega_2 - \frac{1}{M_2} g(\delta_1, \delta_2, \delta_3) \quad (11)$$

$$\dot{\delta}_3 = \frac{1}{M_3} M_3 \omega_3 \quad (12)$$

$$\dot{\omega}_3 = -\frac{D_3}{M_3^2} M_3 \omega_3 - \frac{1}{M_3} h(\delta_1, \delta_2, \delta_3) \quad (13)$$

Here,

$$f(\delta_1, \delta_2, \delta_3) = -\{P_1 - K_1 V_1 \sin \delta_1 - K_2 \sin(\delta_1 - \delta_2)\} \quad (14)$$

$$g(\delta_1, \delta_2, \delta_3) = -\{P_2 - K_2 \sin(\delta_2 - \delta_1) - V_2 \sin(\delta_2 - \delta_3)\} \quad (15)$$

$$h(\delta_1, \delta_2, \delta_3) = -\{P_3 - V_2 \sin(\delta_3 - \delta_2)\} \quad (16)$$

The derivation of Lyapunov function for the three-machine infinite-bus power system, equations (8), (9), (10), (11), (12) and (13) could be determined as:

$$\begin{bmatrix} \dot{\delta}_1 \\ \dot{\omega}_1 \\ \dot{\delta}_2 \\ \dot{\omega}_2 \\ \dot{\delta}_3 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{M_1} & 0 & 0 & 0 & 0 \\ \frac{1}{M_1} & \frac{D_1}{M_1^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{M_2} & 0 & 0 \\ 0 & 0 & \frac{1}{M_2} & \frac{D_2}{M_2^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{M_3} \\ 0 & 0 & 0 & 0 & \frac{1}{M_3} & \frac{D_3}{M_3^2} \end{bmatrix} \begin{bmatrix} f(\delta_1, \delta_2, \delta_3) \\ M_1 \omega_1 \\ g(\delta_1, \delta_2, \delta_3) \\ M_2 \omega_2 \\ h(\delta_1, \delta_2, \delta_3) \\ M_3 \omega_3 \end{bmatrix} \quad (17)$$

The equation (17) for the model defined in equation (1) is an alternative definition for dynamics of this model.

For the  $(\delta_{10}, \omega_{10}, \delta_{20}, \omega_{20}, \delta_{30}, \omega_{30})$ 's equilibrium point, a candidate energy function, which is seen on the right of the equation (17), ((6x1) gradient matrix seen on the right of the equation (18)) is obtained and therefore it can be used in equation (7). The candidate energy function can be written in equation (7) as:

$$V(\delta_1, \omega_1, \delta_2, \omega_2, \delta_3, \omega_3) = \int_{(\delta_{10}, \omega_{10}, \delta_{20}, \omega_{20}, \delta_{30}, \omega_{30})}^{(\delta_1, \omega_1, \delta_2, \omega_2, \delta_3, \omega_3)} \begin{bmatrix} f(\delta_1, \delta_2, \delta_3) \\ M_1 \omega_1 \\ g(\delta_1, \delta_2, \delta_3) \\ M_2 \omega_2 \\ h(\delta_1, \delta_2, \delta_3) \\ M_3 \omega_3 \end{bmatrix}^T \begin{bmatrix} d\delta_1 \\ d\omega_1 \\ d\delta_2 \\ d\omega_2 \\ d\delta_3 \\ d\omega_3 \end{bmatrix} \quad (18)$$

If  $f(\delta_1, \delta_2, \delta_3)$ ,  $g(\delta_1, \delta_2, \delta_3)$  and  $h(\delta_1, \delta_2, \delta_3)$  are replaced on equation (18), energy function of the three-machine infinite-bus power system is obtained as:

$$V(\delta_1, \omega_1, \delta_2, \omega_2, \delta_3, \omega_3) = \left. \begin{aligned} &0.31112 P_1 + K_1 V_1 \cos(0.31112) \\ &+ K_2 \cos(\delta_2 - 0.31112) - P_1 \delta_1 - K_1 V_1 \cos(\delta_1) \\ &- K_2 \cos(\delta_1 - \delta_2) + 0.51248 P_2 + K_2 \cos(\delta_1 - 0.51248) \\ &+ V_2 \cos(\delta_3 - 0.51248) - P_2 \delta_2 - K_2 \cos(\delta_1 - \delta_2) \\ &- V_2 \cos(\delta_2 - \delta_3) + 0.61366 P_3 + V_2 \cos(\delta_2 - 0.61366) \\ &- P_3 \delta_3 - V_2 \cos(\delta_2 - \delta_3) - \frac{1}{2} M_1 \omega_1^2 - \frac{1}{2} M_2 \omega_2^2 - \frac{1}{2} M_3 \omega_3^2 \end{aligned} \right\} \quad (19)$$

The equilibrium point is  $(\delta_1^*, \omega_1^*, \delta_2^*, \omega_2^*, \delta_3^*, \omega_3^*) = (0.31112, 0, 0, 0.51248, 0, 0, 0.61366, 0, 0)$  [17].

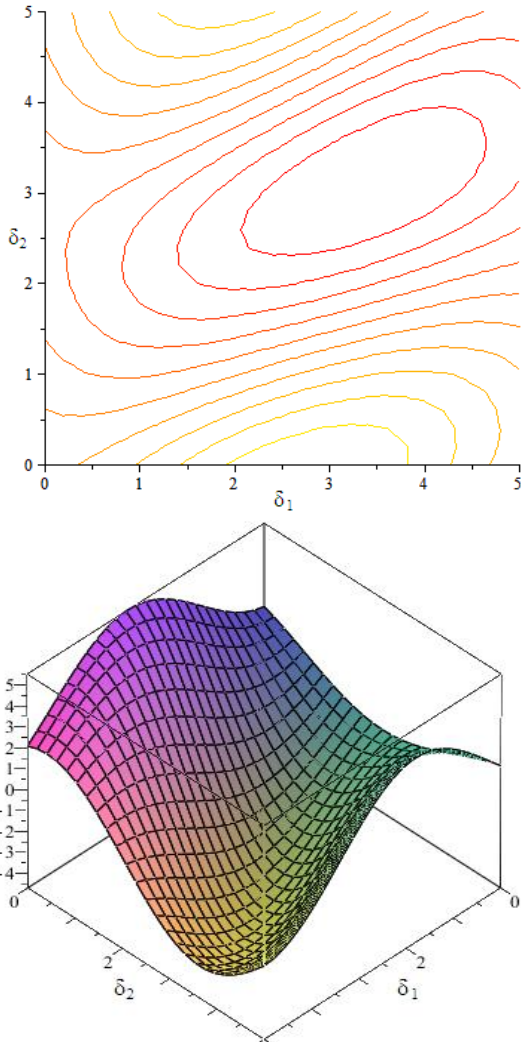
## NUMERICAL SIMULATIONS

A system consisting of three-machine infinite-bus power system already considered in [17, 19], is used as shown in Fig. 1. The parameters of the studied power system are listed in Table 1.

The numerical simulations are carried out by using Maple.

**TABLE 1.** Example power system parameters [17]

$M_1, M_2, M_3$	0.5	$\delta_1$	2.60
$D_1, D_2, D_3$	0.005	$\delta_2$	1.15
$P_1, P_2, P_3$	0.1	$\delta_3$	2.17
$K_1, K_2$	1.0	$\omega_1, \omega_2, \omega_3$	0.0
$V_1$	0.98	$V_2$	0.99



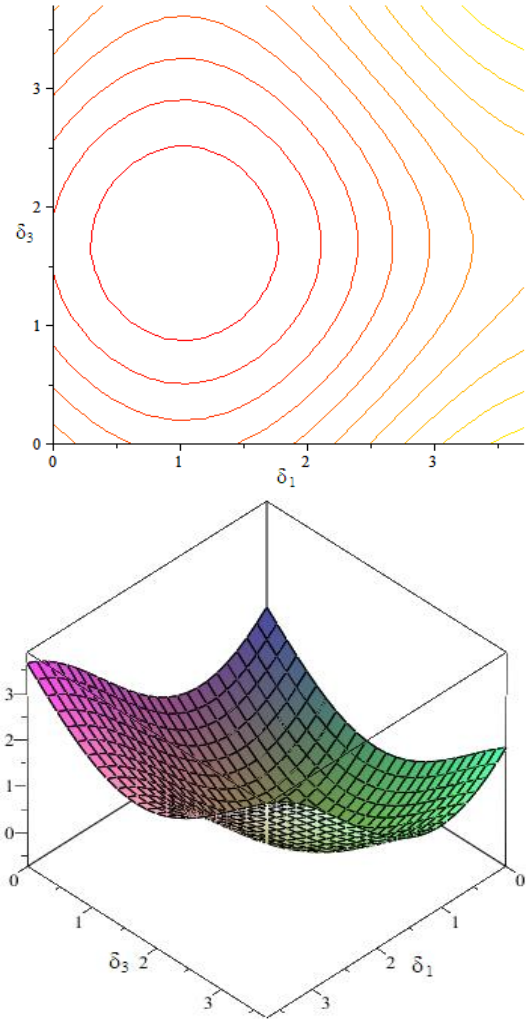
**FIGURE 2.** Stored energy for  $\delta_3=2,17$ : (a) plot of the contours (b) 3D of the studied model

The energy density of the studied system is in the range of  $1,5 \leq \delta_1 \leq 4$  and  $0 \leq \delta_2 \leq 0,5$ , which is seen as in

Fig. 2 and Table 2. The energy density of the studied system varies between 3,8 and 5,5 energy units around these points.

**TABLE 2.** Energy measurement of  $\delta_1-\delta_2$

$\delta_2$	Energy										
0	1,5	2,0	2,8	3,8	4,8	5,4	5,5	5,1	4,3	3,2	2,1
0,5	1,0	1,0	1,4	2,1	3,0	3,8	4,4	4,4	4,1	3,3	2,3
1	0,4	-0,1	-0,2	0,2	0,8	1,6	2,3	2,8	2,9	2,6	1,9
1,5	-0,2	-0,9	-1,4	-1,6	-1,4	-0,9	-0,2	0,5	1,0	1,1	0,9
2	-0,6	-1,4	-2,1	-2,7	-3,0	-2,9	-2,5	-1,9	-1,2	-0,6	-0,3
2,5	-0,7	-1,3	-2,1	-3,0	-3,7	-4,1	-4,1	-3,7	-3,1	-2,3	-1,6
3	-0,6	-0,8	-1,4	-2,3	-3,3	-4,1	-4,6	-4,7	-4,3	-3,5	-2,7
3,5	-0,2	0,1	-0,2	-0,8	-1,9	-3,0	-3,9	-4,4	-4,5	-4,0	-3,1
4	0,3	1,0	1,3	1,0	0,2	-0,9	-2,1	-3,1	-3,6	-3,6	-3,0
4,5	0,8	1,9	2,6	2,8	2,4	1,4	0,2	-1,0	-2,0	-2,4	-2,3
5	1,1	2,3	3,4	4,0	4,1	3,5	2,5	1,3	0,0	-0,8	-1,2
$\delta_1$	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5



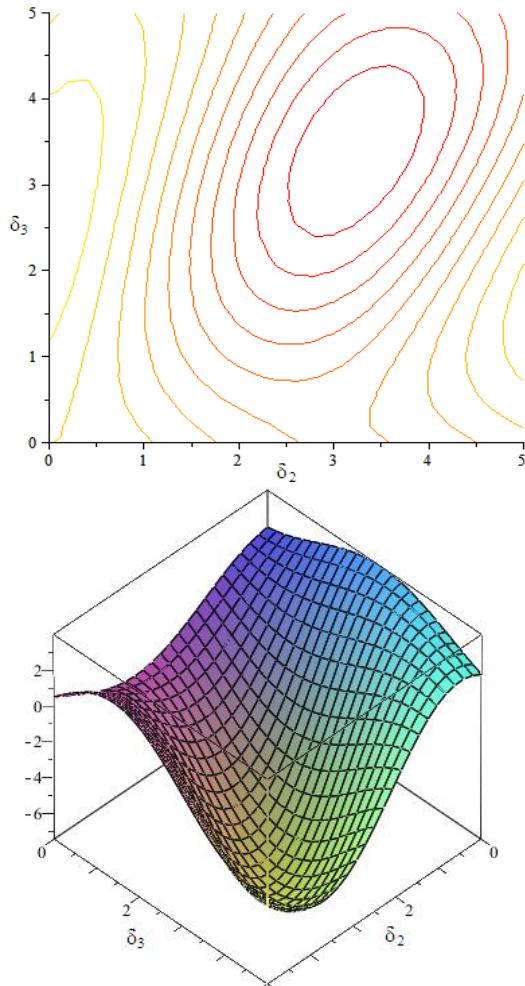
**FIGURE 3.** Stored energy for  $\delta_2=1,15$ : (a) plot of the contours (b) 3D of the studied model

The energy density of the studied system is in the range of  $3,5 \leq \delta_1 \leq 4,5$  and  $3,5 \leq \delta_2 \leq 5$ , which is seen as in

Fig. 3 and Table 3. The energy density of the studied system varies between 3,5 and 4,5 energy units around these points.

**TABLE 3.** Energy measurement of  $\delta_1$ - $\delta_3$

$\delta_3$	Energy										
0	1,6	1,0	0,8	1,0	1,5	2,2	3,0	3,5	3,7	3,6	3,1
0,5	0,9	0,3	0,1	0,3	0,8	1,5	2,3	2,8	3,1	2,9	2,4
1	0,4	-0,2	-0,4	-0,3	0,3	1,0	1,7	2,3	2,5	2,3	1,8
1,5	0,1	-0,5	-0,7	-0,5	0,0	0,7	1,4	2,0	2,2	2,1	1,5
2	0,1	-0,4	-0,7	-0,5	0,0	0,7	1,5	2,0	2,3	2,1	1,6
2,5	0,5	-0,1	-0,3	-0,2	0,4	1,1	1,8	2,4	2,6	2,4	1,9
3	1,0	0,4	0,2	0,4	0,9	1,6	2,4	2,9	3,2	3,0	2,5
3,5	1,6	1,0	0,8	1,0	1,5	2,2	3,0	3,5	3,8	3,6	3,1
4	2,1	1,5	1,3	1,5	2,0	2,7	3,5	4,0	4,3	4,1	3,6
4,5	2,4	1,8	1,6	1,8	2,3	3,0	3,7	4,3	4,5	4,4	3,8
5	2,3	1,8	1,5	1,7	2,2	3,0	3,7	4,2	4,5	4,3	3,8
$\delta_1$	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5



**FIGURE 4.** Stored energy for  $\delta_1=2,6$ : (a) plot of the contours (b) 3D of the studied model

The energy density of the studied system is in the range of  $0 \leq \delta_2 \leq 0,5$  and  $1,5 \leq \delta_2 \leq 4$ , which is seen as in

Fig. 4 and Table 4. The energy density of the studied system varies between 4,9 and 5,6 energy units around these points.

**TABLE 4.** Energy measurement of  $\delta_2$ - $\delta_3$

$\delta_3$	Energy										
0	3,5	3,2	2,6	1,8	1,1	0,5	0,2	0,3	0,8	1,4	2,1
0,5	3,8	3,0	2,0	1,0	0,2	-0,2	-0,1	0,5	1,4	2,4	3,2
1	4,3	3,1	1,6	0,1	-0,9	-1,3	-1,0	0,0	1,3	2,8	3,9
1,5	4,9	3,4	1,5	-0,5	-2,0	-2,6	-2,3	-1,2	0,6	2,5	4,0
2	5,3	3,8	1,6	-0,8	-2,7	-3,8	-3,8	-2,6	-0,7	1,6	3,6
2,5	5,6	4,3	2,0	-0,6	-3,0	-4,6	-5,0	-4,1	-2,2	0,3	2,7
3	5,5	4,6	2,5	-0,1	-2,8	-4,8	-5,7	-5,2	-3,5	-1,1	1,5
3,5	5,2	4,7	3,1	0,6	-2,1	-4,4	-5,7	-5,7	-4,5	-2,3	0,3
4	4,6	4,6	3,4	1,4	-1,1	-3,4	-5,0	-5,5	-4,7	-2,9	-0,6
4,5	3,9	4,3	3,6	2,0	-0,1	-2,2	-3,8	-4,6	-4,2	-3,0	-1,0
5	3,4	3,8	3,4	2,2	0,6	-1,1	-2,5	-3,3	-3,2	-2,3	-0,9
$\delta_2$	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5

For positive values of the candidate energy function which is obtained from Eq. (19), the system is stable, for negative values, the system is unstable in the sense of Lyapunov (Table 2, Table 3 and Table 4).

## CONCLUSION

In this study, the variable gradient method for the construction of a Lyapunov (energy) function for the three-machine infinite-bus power system model with the governors and automatic voltage regulators removed has been presented. The variable gradient method is a general and in principle applicable way for solving the problems of power system stability. This method can be applied to obtain a new Lyapunov function in the case of the multi-machine power systems. Hopefully, this work helps to the construction of a suitable Lyapunov (energy) function for the problem of power system stability.

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