Stability Region Estimation for Two-Machine Infinite-Bus Power System using Energy Function

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Abstract. Lyapunov stability theory is a general and successful approach. Despite this, there is no general method for obtaining Lyapunov function. Some approaches to constructing Lyapunov functions are suitable. In this work, we focus on providing a suitable candidate Lyapunov (energy) function by using the variable gradient method based on Lyapunov's second method. In addition, the stability region of the energy function is analyzed and visualized for two-machine infinite-bus power system. The results show that energy function can be used to obtain best estimates of the stability region.

Keywords: Two-machine infinite-bus power system, Lyapunov (energy) function, Stability region, Variable gradient method
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INTRODUCTION

Power system stability has in recent years become an important problem over a wide range of operating conditions. There is a large amount of literature on the stability of electric power systems. Power system stability is defined as a power system at a given initial operating state and subjected to a given disturbance is stable if the system state stays within determined bounds and the system comes into a new stable state of equilibrium within a determined period of time [1].

In general, an electric power system is expressed with nonlinear dynamical equations including of system parameters. Any change in any parameter of the system could affect the behavior of nonlinear dynamical system. Hence, the system parameter variations can cause system instability. Lyapunov's second method (also called Lyapunov's direct method) provides an important tool for the stability analysis of a system without solving differential equations. Lyapunov (energy) functions offer a certain estimate of the stability region. Lyapunov stability theory is a general and successful approach. Despite this, there exists no general method for obtaining Lyapunov function. Some systematic approaches to constructing Lyapunov functions are Schultz and Gibson's variable gradient method [2], Ingwerson's method [3], Krasovski's method [4], Zubov's method [5] and Energy-Casimir method [6]. We focus on the variable gradient method, which provides a formal Lyapunov function design. Additional studies of energy (Lyapunov) function and construction of Lyapunov function can be found in [7-16], and the references cited therein.

Kopell and Washburn [17] used Melkinov's method for detecting chaotic motions in the two-degree-of-freedom swing equations. A simulation of a two machine power system, each nonzero damping has been presented in [18]. Ueda et al. [19] studied coupled swing equations modeling the dynamics of two electric generators connected to an infinite bus by a simple transmission network. They concluded that the attractor-basin phase portrait provides the most direct information about system stability. The concepts of transiently chaotic swings and windowed Lyapunov exponents to power system dynamics has been introduced by Liu et al. in [20], the proposed method being applied to a three machine system with lossless transmission lines therein. The two degrees of freedom swing equation system has been investigated as the model of electric power system to analyze its transient stability problem, and chaotic behavior and global basin structure have also been studied in [21, 22]. Extended Lyapunov functions are proposed for a single-machine infinite-bus system considering line losses in [23]. See also [24], where authors detected an extended Lyapunov function for power systems incorporating the transfer conductances by extending the invariance principle. Recently, the transient stability of a two machine infinite bus system with or without decentralized nonlinear controller has been presented when the system was affected by large disturbances, by comparison of time domain approach in [25].

The main purpose of this paper to visualize the stability region of the energy function for two-machine infinite-bus power system by using the variable gradient method based on Lyapunov's second method. The simulation results are obtained by using MAPLE [26] software.

The remainder of this paper is organized as follows: The next section presents power system model of a two-machine infinite-bus. The main idea of the variable gradient method for constructing a suitable Lyapunov function is outlined in section 3. In section 4, gives a candidate Lyapunov (energy) function of the two-machine infinite-bus power system model. Numerical simulation is given in section 5. The last section is a conclusion.

POWER SYSTEM MODEL

Consider the power system shown in Fig. 1 that three-machine power system [20]. The machine 3 is treated as infinite bus.

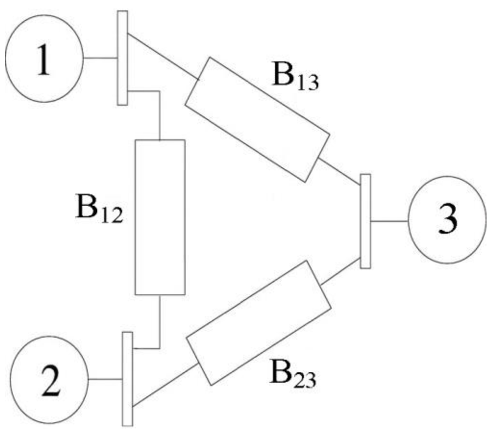

FIGURE 1. Power system model [20]

The two-machine infinite-bus power system can be mathematically described by the following pair of differential equations:

$$\left.\begin{aligned} \dot{\delta}_1 &= \omega_1 \\ \dot{\omega}_1 &= P_1 - B_{13}\sin(\delta_1) - D_1\omega_1 - B_{12}\sin(\delta_1 - \delta_2) \\ \dot{\delta}_2 &= \omega_2 \\ \dot{\omega}_2 &= P_2 - B_{23}\sin(\delta_2) - D_2\omega_2 - B_{21}\sin(\delta_2 - \delta_1) \end{aligned}\right\} \quad (1)$$

where δ_i is the rotor angle of the i^{th} machine with respect to a synchronously rotating reference frame, ω_i is the angular speed of the i^{th} machine, P_i is the mechanical input power, D_i is damping coefficient of the *i*th machine and B_{12}, B_{13}, B_{21} and B_{23} are line parameters.

THE VARIABLE GRADIENT METHOD

A brief review of the variable gradient method [12, 13, 15] will be presented. Consider the nonlinear system

$$\dot{x} = f(x,t) \quad (2)$$

Denote a test Lyapunov function by using V and its time derivative $\dot{V}$. In (2), V is x's open function but not t's.

$$\dot{V} = \frac{\partial V}{\partial x_1}\dot{x}_1 + \frac{\partial V}{\partial x_2}\dot{x}_2 + \ldots + \frac{\partial V}{\partial x_n}\dot{x}_n \quad (3)$$

Denoted from the point of the gradient, equation (3) can be written as:

$$\dot{V} = (\nabla V)^* \dot{x} \quad (4)$$

where $(\nabla V)^*$ is the transpose of ∇V. V is obtained as a line integral of ∇V [27], as:

$$V = \int_0^x (\nabla V)^* dx \quad (5)$$

The integral's upper limit here does not point that V is a vector quantity, but integral is preferred to line integral of a random point $(x_1, x_2, \ldots, x_n)$ at the phase-space. This integral can be made independent of integration method.

A gradient system of the form [28]

$$\dot{x} = -A\nabla V(x, x_0) \quad (6)$$

V: $\Re^n x \Re^n \to \Re$ is a continuously differentiable function. $A \in \Re^{nxn}$ is defined as det(A)≠0 and $V(x,x_0)=0$ at $x=x_0$.

If the Hessian of $V(x,x_0)$ is completely positive definite at x_0, the equilibrium point is asymptotically stable at x_0.

Lyapunov function is given as:

$$V(x) = \int_{x_0}^{x} \left[f(\xi)\right]^T d\xi \quad (7)$$

The equation (7) will be used in order to construct a suitable candidate Lyapunov function of the two-machine infinite-bus power system.

ENERGY FUNCTION FOR THE TWO-MACHINE INFINITE BUS POWER SYSTEM

The two-machine infinite-bus power system model differential equations above can be written again under the condition that the generator mechanical power is equivalent to the active load requirement ($P_m = P_l$).

$$\dot{\delta}_1 = \omega_1 \tag{8}$$

$$\dot{\omega}_1 = -D_1\omega_1 + f(\delta_1, \delta_2) \tag{9}$$

$$\dot{\delta}_2 = \omega_2 \tag{10}$$

$$\dot{\omega}_2 = -D_2\omega_2 + g(\delta_1, \delta_2) \tag{11}$$

Here,

$$f(\delta_1,\delta_2) = P_1 - B_{13}\sin(\delta_1) - B_{12}\sin(\delta_1 - \delta_2) \tag{12}$$

$$g(\delta_1,\delta_2) = P_2 - B_{23}\sin(\delta_2) - B_{21}\sin(\delta_2 - \delta_1) \tag{13}$$

The derivation of Lyapunov function for the two-machine infinite-bus power system, equations (8), (9), (10) and (11) could be determined as:

$$\begin{bmatrix}\delta_1\\ \omega_1\\ \delta_2\\ \omega_2\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 1 & -D_1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & -D_2\end{bmatrix}\begin{bmatrix}f(\delta_1,\delta_2)\\ \omega_1\\ g(\delta_1,\delta_2)\\ \omega_2\end{bmatrix} \tag{14}$$

The equation (14) for the model defined in equation (1) is an alternative definition for dynamics of this model.

For the (δ_{10}, w_{10}, δ_{20}, w_{20}, δ_{30}, w_{30})'s equilibrium point, a candidate energy function, which is seen on the right of the equation (14), ((4x1) gradient matrix seen on the right of the equation (15)) is obtained and therefore it can be used in equation (7). The candidate energy function can be written in equation (7) as:

$$V(\delta_1,\omega_1,\delta_2,\omega_2) = \int_{(\delta_{10},\omega_{10},\delta_{20},\omega_{20})}^{(\delta_1,\omega_1,\delta_2,\omega_2)} \begin{bmatrix}f(\delta_1,\delta_2)\\ \omega_1\\ g(\delta_1,\delta_2)\\ \omega_2\end{bmatrix}\begin{bmatrix}d\delta_1\\ d\omega_1\\ d\delta_2\\ d\omega_2\end{bmatrix} \tag{15}$$

If $f(\delta_1,\delta_2)$ and $g(\delta_1,\delta_2)$ are replaced on equation (15), energy function of the two-machine infinite-bus power system is obtained as:

$$\begin{aligned} V(\delta_1,\omega_1,\delta_2,\omega_2) &= \delta_1 P_1 + B_{13}\cos(\delta_1) - B_{13}\\ &+ B_{12}\cos(\delta_2-\delta_1) - B_{12}\cos(\delta_2) + \frac{1}{2}\omega_1^{\,2}\\ &+ \delta_2 P_2 + B_{23}\cos(\delta_2) - B_{23} + B_{21}\cos(\delta_2-\delta_1)\\ &- B_{21}\cos(\delta_1) + \frac{1}{2}\omega_2^{\,2} \end{aligned} \tag{16}$$

NUMERICAL SIMULATIONS

A system consisting of two-machine infinite-bus power system already considered in [18, 20], is used as shown in Fig. 1. The parameters of the studied power system are set to be P_1=0.59739, B_{13}=1.0, D_1=0.5, P_2=0.89739, B_{23}=1.0, D_2=0.3 and B_{12}=B_{21}=0.1 [18, 20]. The numerical simulations are carried out by using Maple.

Level curves of energy function and stability region estimate (P_1, P_2) = (0.57039, 0.89739), (0.59739, 0.89739), (0.59739, 0.79739), (0.59739, 0.99739) are respectively produced in Figures 2-5.

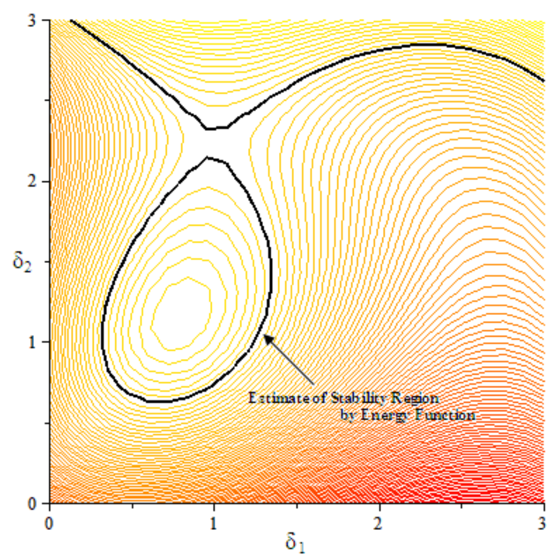

FIGURE 2. Fig.2: Level curves of energy function and estimate of stability region (P_1=0.57039, P_2=0.89739)

Fig. 2 shows the estimate of stability region (solid black line) by the energy function in (16) with P_1=0.57039, P_2=0.89739. The inside of the black solid line is the estimate of stability region by energy function.

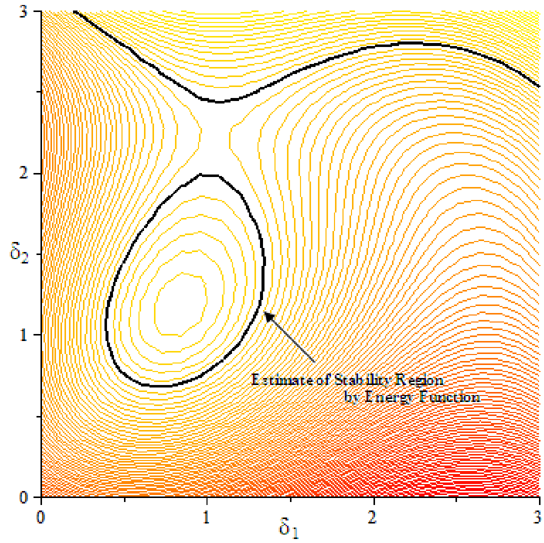

FIGURE 3. Level curves of energy function and estimate of stability region (P_1=0.59739, P_2=0.89739)

Fig. 3 illustrates the estimate of stability region (solid black line) by the energy function in (16) with different mechanical input power P_1. The inside of the black solid line is the estimate of stability region by energy function.

It can be observed that as the mechanical input power P_1 increases, the estimate of stability region becomes smaller in Fig. 3.

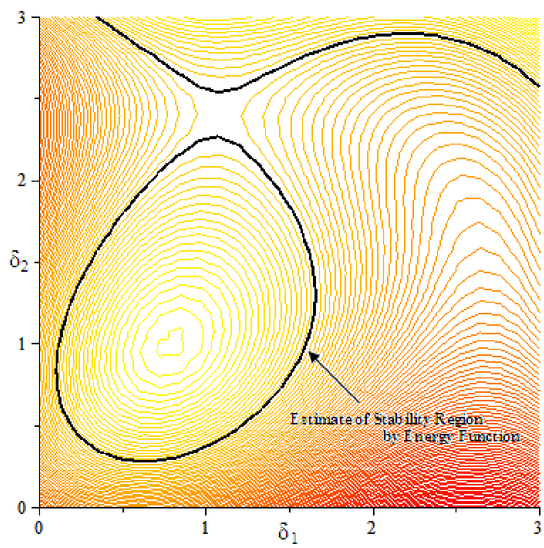

FIGURE 4. Level curves of energy function and estimate of stability region (P_1=0.59739, P_2=0.79739)

Fig. 4 shows the estimate of stability region (solid black line) by the energy function in (16) with P_1=0.59739, P_2=0.79739. The inside of the black solid line is the estimate of stability region by energy function.

Fig. 4 indicates that as the mechanical input power P_2 decreases, the estimate of stability region becomes larger.

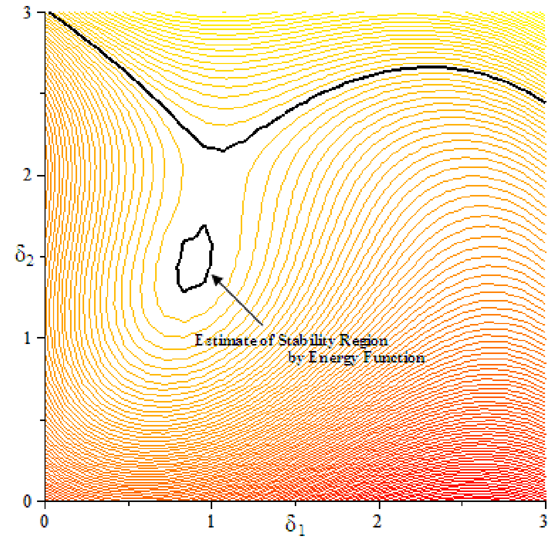

FIGURE 5. Level curves of energy function and estimate of stability region (P_1=0.59739, P_2=0.99739)

Fig. 5 demonstrates the estimate of stability region (solid black line) by the energy function in (16) with different mechanical input power P_2. The inside of the black solid line is the estimate of stability region by energy function.

It can be seen that the mechanical input power P_2 increases, the estimate of stability region becomes smaller in Fig. 5.

CONCLUSION

In this paper, the construction of a Lyapunov (energy) function for the two-machine infinite-bus power system model by using the variable gradient method based on Lyapunov's second method has been presented. And a stability region of the studied power system is estimated using this function in order to study the mechanical input power effect on the estimate of a stability region. The results have shown that energy function can be used for the estimate of stability region.

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